

习题3

1. 选择题

(1)D; (2)B; (3)C; (4)C; (5)D; (6)A; (7)D; (8)C; (9)B; (10)D.

2. 填空题

(1) $\frac{1}{6}f(3x^2 + 1) + C$; (2) $f(1) = e^{-1}$; (3) $\frac{x^3}{3} + \frac{1}{3}(x^2 - 1)^{\frac{3}{2}} + C$;

(4) $\frac{1}{2}\arctan x^2 + \frac{1}{4}\ln(1 + x^4) + C$; (5) $-\frac{4}{3}\sqrt{1 - x\sqrt{x}} + C$; (6) $\frac{1}{12}$;

(7) $f(x) = x^2 + \frac{1}{3}e^{1-x}$; (8) $2\ln(1 + \pi)$; (9) $\sqrt{2}x + y = \frac{\sqrt{2\pi}}{2}$; (10) $2\sqrt{2}$.

3. 计算下列不定积分:

(1) $\frac{1}{2\sqrt{2}}\ln\left|\frac{x + \frac{1}{x} - \sqrt{2}}{x + \frac{1}{x} + \sqrt{2}}\right| + C$; (2) $\frac{1}{2}(\arcsin x)^2 - \frac{\sqrt{1-x^2}}{x}\arcsin x + \ln|x| + C$;

(3) $-\frac{\ln x}{1+x} + \ln\left|\frac{x}{1+x}\right| + C$; (4) $\frac{1}{5}\ln|\sec x + \tan x| - \frac{3}{10}\arctan\left(\frac{1}{2}\tan\frac{x}{2}\right) + C$;

(5) $x^2 - x + \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln(x^2 + x + 1) + \frac{\sqrt{3}}{3}\arctan\frac{2x+1}{\sqrt{3}} + C$;

(6) $\frac{x}{2} + \sqrt{x} - \frac{1}{2}\sqrt{x(1+x)} - \frac{1}{2}\ln(\sqrt{x} + \sqrt{1+x}) + C$;

(7) $\int (\sin x + \cos x)^n \cos 2x dx = \begin{cases} \ln|\sin x + \cos x| + C, & n = -2, \\ \frac{1}{n+2}(\sin x + \cos x)^{n+2} + C, & n \neq -2. \end{cases}$

(8) $\frac{1}{a^2 + b^2}(ax - b\ln|a\sin x + b\cos x|) + C$.

4. $f(x) = \frac{1}{2}\tan(x - \frac{\pi}{4}) + \frac{1}{2} = \frac{\tan x}{1 + \tan x}$.

5. $f(x) = \begin{cases} \frac{1}{3}x^3 + C, & x \geq 0, \\ 1 - \cos x + C, & x < 0. \end{cases}$

6. $f(x) = \frac{xe^{\frac{x}{2}}}{2(1+x)^{\frac{3}{2}}}$.

7. (1) $-\frac{1}{6}$; (2) $\pi(e-1)$; (3) $\begin{cases} \frac{x^2}{2} + x + \frac{1}{2}, & -1 \leq x \leq 0, \\ -\frac{x^2}{2} + x + \frac{1}{2}, & x > 0. \end{cases}$;

(4) $\frac{\pi}{4} + \frac{1}{2}\ln 2$; (5) $2(1 - e^{-1})$

8. $\frac{\pi^2}{4}$. 9. $f(0)$. 10. 2. 11. 证明略. 12. 证明略. 13. 证明略.