习题 3.1

1. A. 2. C. 3. D.

4. (1) $\frac{2}{7}x^{\frac{7}{2}}-5\cdot\frac{2}{3}x^{\frac{3}{2}}+C$; (2) $3\arctan x-2\arcsin x+C$;

(3) $\frac{1}{2}x^2-3x+3\ln|x|+\frac{1}{x}+C$; (4) $\frac{2^x e^x}{1+\ln 2}-3\sin x+C$;

(5) $-\frac{1}{x}-\arctan x+C$; (6) $x^3+\arctan x+C$; (7) $\tan x-x+C$;

(8) $\frac{1}{2}(x-\sin x)+C$; (9) $-4\cot x+C$; (10) e^x+x+C;

(11) $-\cot x-x+C$; (12) $2\arcsin x+C$.

5. $-\frac{1}{x\sqrt{1-x^2}}$.

6. $\begin{cases} 2x-\frac{1}{2}x^2+C_1-4, & x<2, \\ \frac{1}{2}x^2-2x+C_1, & x\geqslant 2. \end{cases}$

7. $f(x)=\ln|x|+1$.

8. $t=50$ 秒, $S=500$ 米.

9. (1) $7x+C$; (2) $-x^2+C$; (3) $3x^4+C$; (4) $e^{2x}+C$; (5) $5\ln|x|+C$; (6) $\sin^2 x+C$; (7) $\sqrt{t}+C$; (8) $\tan 2x+C$.

10. (1) $\frac{1}{3}\sec^3 x-\sec x+C$; (2) $-\frac{1}{5}\cos^5 x+\frac{1}{7}\cos^7 x+C$;

(3) $\frac{1}{2}\ln(1+x^2)+\frac{2}{5}(\arctan x)^{\frac{5}{2}}+C$; (4) $\frac{1}{4}\ln|x|-\frac{1}{12}\ln|4+x^3|+C$;

(5) $-\cot\frac{x}{2}-x+C$; (6) $-\frac{1}{x\ln x}+C$; (7) $-\frac{1}{2}\ln|3-2x|+C$;

(8) $2\sin\sqrt{t}+C$; (9) $\ln|\ln\ln x|+C$; (10) $-\ln\left|\cos\sqrt{1+x^2}\right|+C$;

(11) $\ln|\csc 2x-\cot 2x|+C$; 或 $\ln|\tan x|+C$; (12) $\arctan e^x+C$;

(13) $\frac{1}{2}\arcsin\left(\frac{2x}{3}\right)+\frac{1}{4}\sqrt{9-4x^2}+C$;

(14) $-\frac{1}{97}\frac{1}{(x-1)^{97}}-\frac{1}{49}\frac{1}{(x-1)^{98}}-\frac{1}{99}\frac{1}{(x-1)^{99}}+C$;

(15) $\arctan(x\ln x)+C$; (16) $\tan x+\frac{2}{3}\tan^3 x+\frac{1}{5}\tan^5 x+C$; (17) $\frac{1}{10}\sin 5x+\frac{1}{2}\sin x+C$.

11. $f(x)=2\sqrt{1+x}-1$.

12. (1) $\arcsin x-\frac{x}{1+\sqrt{1-x^2}}+C$ 或 $=\arcsin x-\frac{1-\sqrt{1-x^2}}{x}+C$;

(2) $\sqrt{x^2-9}-3\arccos\frac{3}{|x|}+C$; (3) $\frac{x}{\sqrt{1+x^2}}+C$; (4) $\frac{x}{a^2\sqrt{a^2+x^2}}+C$;

(5) $\frac{1}{2}\ln\left|\sqrt{x^4+1}+x^2\right|+\frac{1}{2}\ln\left|\frac{\sqrt{x^4+1}-1}{x^2}\right|+C$; (6) $\frac{9}{2}\arcsin\frac{x+2}{3}+\frac{x+2}{2}\sqrt{5-4x-x^2}+C$.

13. (1) $\frac{1}{4}x^4\ln x-\frac{1}{16}x^4+C$; (2) $\frac{x}{2}[\sin(\ln x)+\cos(\ln x)]+C$;

(3) $2x\sqrt{e^x-1}-4\sqrt{e^x-1}+4\arctan\sqrt{e^x-1}+C$; (4) $\frac{e^x}{5}(5\sin^2 x-\sin 2x+2\cos 2x)+C$;

(5) $x\ln(1+x^2)-2x+2\arctan x+C$; (6) $-\frac{2e^{-2x}}{17}\left(4\sin\frac{x}{2}+\cos\frac{x}{2}\right)+C$;

(7) $\frac{1}{n+1}x^{n+1}\left(\ln x-\frac{1}{(n+1)}\right)+C$; (8) $-e^{-x}\ln(1+e^x)-\ln(1+e^{-x})+C$.

14. $\frac{e^x(x-2)}{x}+C$. 15. $xf^{-1}(x)-F(f^{-1}(x))+C$.

16. (1) $\ln\left|\frac{x}{x-1}\right|-\frac{1}{x-1}+C$; (2) $\frac{2}{5}\ln|1+2x|-\frac{1}{5}\ln(1+x^2)+\frac{1}{5}\arctan x+C$;

(3) $\frac{x^3}{3}-\frac{1}{2\sqrt{2}}\ln\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right|+C$; (4) $\frac{\sqrt{2}}{8}\ln\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}+\frac{\sqrt{2}}{4}\left(\arctan\frac{\sqrt{2}x}{1-x^2}\right)+C$.

17. (1) $\frac{2}{\sqrt{3}}\arctan\left(\frac{2\tan\frac{x}{2}+1}{\sqrt{3}}\right)+C$; (2) $\ln\left|1+\tan\frac{x}{2}\right|+C$; (3) $\frac{\sqrt{3}}{6}\arctan\left(\frac{2}{\sqrt{3}}\tan x\right)+C$;

(4) $\ln\frac{1+\cos x}{\cos x}+C$; (5) $\frac{1}{2}\left[\ln|1+\tan x|-\frac{1}{2}\ln(1+\tan^2 x)+x\right]+C$;

(6) $\frac{3}{2}\sqrt[3]{(1+x)^2}-3\sqrt[3]{1+x}+3\ln\left|\sqrt[3]{1+x}+1\right|+C$;

(7) $2\sqrt{x}-4\sqrt[4]{x}+4\ln(1+\sqrt[4]{x})+C$; (8) $-\frac{3}{2}\sqrt[3]{\frac{x+1}{x-1}}+C$.