

## 习题4.6

1. 正弦级数:  $x + 1 = \frac{2}{\pi} \left[ (\pi + 2) \sin x - \frac{\pi}{2} \sin 2x + \frac{1}{3} (\pi + 2) \sin 3x - \frac{\pi}{4} \sin 4x + \dots \right], (0 \leq x \leq \pi)$

或余弦级数:  $x + 1 = \frac{\pi + 2}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{(2n-1)^2}, (0 \leq x \leq \pi)$

2.  $f(x) = \cos \frac{x}{2} = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\pi} \frac{1}{n^2 - \frac{1}{4}} \cos nx.$

3.  $M(x) = \frac{2pl}{\pi^2} \left( \sin \frac{\pi x}{l} - \frac{1}{3^2} \sin \frac{3\pi x}{l} + \frac{1}{5^2} \sin \frac{5\pi x}{l} - \dots \right) (0 \leq x \leq l).$

4. 取  $x = \frac{\pi}{3}$  即得要证等式.

5. 1;

6.  $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx = 1 - \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx, 0 \leq x \leq \pi, \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}.$

7.  $\sin \frac{x}{2} = \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} \cos nx, x \in [0, \pi]; \frac{1}{2}.$