

## 习题 5

1. (1)  $(x - x_0, y - y_0, z - z_0), (x_0, y_0, z_0)$ ; (2) 线性相关; (3) 3.

2.  $(0, 2, 0)$ .

3.  $\sqrt{30}$ .

4.  $\overrightarrow{AD} = \frac{1}{2}\mathbf{a} + \mathbf{c}, \overrightarrow{BE} = \mathbf{a} + \frac{1}{2}\mathbf{b}, \overrightarrow{CF} = \mathbf{b} + \frac{1}{2}\mathbf{c}$ ,

证明: 因为  $\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB} = \mathbf{0}$ ,

所以  $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}, \overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \frac{1}{2}(\mathbf{a} + \mathbf{b} + \mathbf{c})$ .

5. 证明: 设  $\triangle ABC$  三边  $\overrightarrow{BC} = \mathbf{a}, \overrightarrow{CA} = \mathbf{b}, \overrightarrow{AB} = \mathbf{c}$  三边中点依次为  $D, E, F, \overrightarrow{EF} = \overrightarrow{EA} + \overrightarrow{AF} = \frac{1}{2}(\mathbf{b} + \mathbf{c}) = -\frac{1}{2}\mathbf{a} = -\frac{1}{2}\overrightarrow{BC}$ , 所以  $\overrightarrow{EF} \parallel \overrightarrow{BC}$  且  $|\overrightarrow{EF}| = \frac{1}{2}|\overrightarrow{BC}|$  即  $EF \parallel BC$  且  $EF = \frac{1}{2}BC$  结论成立.

6. 1.      7.  $\arccos \frac{2}{\sqrt{7}}$ .      8.  $\frac{\pi}{3}$ .      9.  $-4, \frac{\pi}{4}$ .

10. 30.

11.  $(14, 10, 2)$ .

12. 证明: 令  $k_1\mathbf{a} + k_1\mathbf{b} + k_1\mathbf{c} = \mathbf{0}$ , 联立方程组求得一组解:  $(5, 1, -1)$ ,

所以  $5\mathbf{a} + \mathbf{b} - \mathbf{c} = \mathbf{0}, \mathbf{c} = 5\mathbf{a} + \mathbf{b}$ .

13.  $x^2 + y^2 - 2x + 2y - 4z + 6 = 0$ .

14. (1) 母线:  $\Gamma \begin{cases} z = y^2 \\ x = 0 \end{cases}$  旋转轴  $z$  轴.

(2) 母线:  $\Gamma \begin{cases} z = \sqrt{3}y \\ x = 0 \end{cases}$  旋转轴  $z$  轴.

15.  $x \pm \sqrt{26}y + 3z - 3 = 0$ .

16.  $x + 2y + 1 = 0$ .

17.  $\frac{x+1}{4} = \frac{y}{19} = \frac{z-4}{7}$ .

18.  $(0, 0, \frac{1}{5})$ , 最小值为  $\sqrt{\frac{6}{5}}$ .

19.  $xOy$  面:  $\begin{cases} x^2 + y^2 - x - y = 0 \\ z = 0 \end{cases}$ ;  $yOz$  面:  $\begin{cases} z^2 + 2y^2 + 2yz - 4y - 3z + 2 = 0 \\ x = 0 \end{cases}$ ;

$zOx$  面:  $\begin{cases} 2x^2 + z^2 + 2xz - 3z - 4x + 2 = 0 \\ y = 0 \end{cases}$ .

20.  $xOy$  面:  $\begin{cases} (x-1)^2 + y^2 \leq 1 \\ z = 0 \end{cases}$ ;  $yOz$  面:  $\begin{cases} (z-2)^2 + 4y^2 \leq 4 \\ x = 0 \end{cases}$ ;  $zOx$  面:  $\begin{cases} x \leq z \leq \sqrt{2x} \\ y = 0 \end{cases}$ .

21. 略.