习题5.3

1. (1)$\boldsymbol{a}\cdot\boldsymbol{b}=3$，$\boldsymbol{a}\times\boldsymbol{b}=(5,1,7)$；
(2)$(-2\boldsymbol{a})\cdot 3\boldsymbol{b}=-18$，$\boldsymbol{a}\times 2\boldsymbol{b}=(10,2,14)$；
(3)$\cos(\widehat{\boldsymbol{a},\boldsymbol{b}})=\dfrac{\sqrt{21}}{14}$.

2. $-\dfrac{3}{2}$.

3. $\pm\left(\dfrac{2}{\sqrt{6}},-\dfrac{1}{\sqrt{6}},-\dfrac{1}{\sqrt{6}}\right)$.

4. 5880 J.

5. $|\overrightarrow{F_1}|\cdot x_1\sin\theta_1=|\overrightarrow{F_2}|\cdot x_2\sin\theta_2$.

6. $\dfrac{10}{3}$.

7. $\lambda=2\mu$.

8. 设圆心为O，r半径，AB为是直径，C是圆上一点，只需证明向量$\overrightarrow{AC}\cdot\overrightarrow{BC}=0$，设$\overrightarrow{OC}=(x,y)$，则$|\overrightarrow{OC}|^2=x^2+y^2=r^2$，$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=(x+r,y)$，
$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=(x-r,y)$，$\overrightarrow{AC}\cdot\overrightarrow{BC}=(x+r)(x-r)+y^2=x^2-r^2+y^2=0$，从而$\overrightarrow{AC}\perp\overrightarrow{BC}$.

9. (1)$-8\boldsymbol{j}-24\boldsymbol{k}$；(2)$-\boldsymbol{j}-\boldsymbol{k}$；(3)2.

10. $\dfrac{\sqrt{19}}{2}$.

11. 证明：$$(\boldsymbol{a}\times\boldsymbol{b})\cdot\boldsymbol{c}=c_x\begin{vmatrix}a_y & a_z\\ b_y & b_z\end{vmatrix}-c_y\begin{vmatrix}a_x & a_z\\ b_x & b_z\end{vmatrix}+c_z\begin{vmatrix}a_x & a_y\\ b_x & b_y\end{vmatrix}=\begin{vmatrix}a_x & a_y & a_z\\ b_x & b_y & b_z\\ c_x & c_y & c_z\end{vmatrix}=(\boldsymbol{b}\times\boldsymbol{c})\cdot\boldsymbol{a}.$$
同样的方法可以证明$(\boldsymbol{a}\times\boldsymbol{b})\cdot\boldsymbol{c}=(\boldsymbol{c}\times\boldsymbol{a})\cdot\boldsymbol{b}$，
于是$(\boldsymbol{a}\times\boldsymbol{b})\cdot\boldsymbol{c}=(\boldsymbol{b}\times\boldsymbol{c})\cdot\boldsymbol{a}=(\boldsymbol{c}\times\boldsymbol{a})\cdot\boldsymbol{b}$.

12. $\boldsymbol{V}=\sqrt{2}(2,1,1)$.

13. 不共面.

14. 证明：令$\boldsymbol{a}=(a_1,a_2,a_3)$，$\boldsymbol{b}=(b_1,b_2,b_3)$，
利用$|\boldsymbol{a}||\boldsymbol{b}|\geqslant\boldsymbol{a}\cdot\boldsymbol{b}$，得$\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}\geqslant|a_1b_1+a_2b_2+a_3b_3|$，
取等号当且仅当$\angle(\widehat{\boldsymbol{a},\boldsymbol{b}})=0$即$(a_1,a_2,a_3)=\lambda(b_1,b_2,b_3)(\lambda\in R)$，$\boldsymbol{a}$，$\boldsymbol{b}$共线.