习题 6

1. (1) $D=\{(x,y)\mid x^2+2y^2\neq 0\}$;

(2) $D=\{(x,y)\mid x-2y+1>0\}$;

(3) $D=\{(x,y)\mid \frac{x^2}{a^2}+\frac{y^2}{b^2}\leqslant 1\}$;

(4) $D=\{(x,y)\mid x>y,\ x>-1\}$;

(5) $D=\{(x,y)\mid y\geqslant 0,\ x\geqslant 0,\ x^2\geqslant y\}$;

(6) $D=\{(x,y)\mid x>0,\ -x\leqslant y\leqslant x\}\cup\{(x,y)\mid x<0,\ x\leqslant y\leqslant -x\}$;

(7) $\{(x,y,z)\mid x>0,\ y>0,\ z>0\}$.

2. (1) 1; (2) 0; (3) 1; (4) 6.

3. (1) $\frac{\partial z}{\partial x}=\frac{2}{y\sin\frac{2x}{y}}$, $\frac{\partial z}{\partial y}=\frac{-2x}{y^2\sin\frac{2x}{y}}$;

(2) $\frac{\partial z}{\partial x}=\frac{y}{2\sqrt{x(1-xy^2)}}$, $\frac{\partial z}{\partial y}=\sqrt{\frac{x}{1-xy^2}}$;

(3) $\frac{\partial z}{\partial x}=\frac{y}{x^2}\sin\frac{x}{y}\sin\frac{y}{x}+\frac{1}{y}\cos\frac{y}{x}\cos\frac{x}{y}$, $\frac{\partial z}{\partial y}=-\frac{x}{y^2}\cos\frac{x}{y}\cos\frac{y}{x}-\frac{1}{x}\sin\frac{x}{y}\sin\frac{y}{x}$;

(4) $\frac{\partial z}{\partial x}=-\frac{y}{x^2}3^{\frac{y}{x}}\ln 3$, $\frac{\partial z}{\partial y}=\frac{1}{x}3^{\frac{y}{x}}\ln 3$;

(5) $\frac{\partial z}{\partial x}=y\mathrm{e}^{\sin\pi xy}(1+\pi xy\cos\pi xy)$, $\frac{\partial z}{\partial y}=x\mathrm{e}^{\sin\pi xy}(1+\pi xy\cos\pi xy)$;

(6) $\frac{\partial z}{\partial x}=\frac{1}{x+\ln y}$, $\frac{\partial z}{\partial y}=\frac{1}{y(x+\ln y)}$;

(7) $\frac{\partial z}{\partial x}=\frac{1}{2\sqrt{x}}\sin\frac{y}{x}-\frac{y}{x\sqrt{x}}\cos\frac{y}{x}$, $\frac{\partial z}{\partial y}=\frac{1}{\sqrt{x}}\cos\frac{y}{x}$;

(8) $\frac{\partial u}{\partial t}=\rho\varphi\mathrm{e}^{t\varphi}+1$, $\frac{\partial u}{\partial \rho}=\mathrm{e}^{t\varphi}$, $\frac{\partial u}{\partial \varphi}=\rho t\mathrm{e}^{t\varphi}-\mathrm{e}^{-\varphi}$;

(9) $\frac{\partial u}{\partial \varphi}=\mathrm{e}^{\varphi+\theta}[\cos(\theta-\varphi)+\sin(\theta-\varphi)]$, $\frac{\partial u}{\partial \theta}=\mathrm{e}^{\varphi+\theta}[\cos(\theta-\varphi)-\sin(\theta-\varphi)]$.

4. (1) $\frac{\partial^2 z}{\partial x^2}=\frac{2}{y}\sec^2\frac{x^2}{y}+\frac{8x^2}{y^2}\sec^3\frac{x^2}{y}\sin\frac{x^2}{y}$, $\frac{\partial^2 z}{\partial y^2}=\frac{2x^2}{y^3}\sec^2\frac{x^2}{y}+\frac{2x^4}{y^4}\sec^3\frac{x^2}{y}\sin\frac{x^2}{y}$,

$\frac{\partial^2 z}{\partial x\partial y}=-\frac{2x}{y^2}\sec^2\frac{x^2}{y}-\frac{4x^3}{y^3}\sec^3\frac{x^2}{y}\sin\frac{x^2}{y}$;

(2) $\frac{\partial^2 z}{\partial x^2}=-\frac{3xy^2}{(x^2+y^2)^{\frac{5}{2}}}$, $\frac{\partial^2 z}{\partial y^2}=\frac{x(2y^2-x^2)}{(x^2+y^2)^{\frac{5}{2}}}$.

(3) $\frac{\partial^2 z}{\partial x^2}=\frac{xy^3}{\sqrt{(1-x^2y^2)^3}}$, $\frac{\partial^2 z}{\partial x\partial y}=\frac{1}{\sqrt{(1-x^2y^2)^3}}$.

(4) $\frac{\partial^2 y}{\partial y^2}=\frac{\ln x(\ln x-1)}{y^2}y^{\ln x}$, $\frac{\partial^2 z}{\partial x\partial y}=\frac{(\ln x\ln y+1)}{xy}y^{\ln x}$;

(5) $\dfrac{\partial^3 u}{\partial x\partial y\partial z}=(1+3xyz+x^2y^2z^2)e^{xyz}$.

5. (1) $dz=\left(ye^{xy}+\dfrac{1}{x+y}\right)dx+\left(xe^{xy}+\dfrac{1}{x+y}\right)dy$;

(2) $dz=\dfrac{dx}{1+x^2}+\dfrac{dy}{1+y^2}$;

(3) $dz=(xdy+ydx)\cos(xy)$;

(4) $dz=\dfrac{4xy(xdy-ydx)}{(x^2-y^2)^2}$;

(5) $dz=(2e^{-y}-\dfrac{\sqrt{3}}{2\sqrt{x}})dx-2xe^{-y}dy$;

(6) $du=x^{xy}[y(1+\ln x)dx+x\ln xdy]$;

(7) $du=\dfrac{3dx-2dy+dz}{3x-2y+z}$;

(8) $du=\dfrac{2(x-y)(dx-dy)}{1+(x-y)^4}$.

6. (1) $dz=-4(dx+dy)$; (2) $dz=2dx-dy$.

7. $\Delta z=0.02$, $dy=0.03$

8. $z=xy^2+x^2+3y$.

9. $-\dfrac{9\sqrt{3}}{2}$;

10. $\left.\dfrac{\partial u}{\partial l}\right|_{(x_0,y_0,z_0)}=x_0+y_0+z_0$ 处沿球面在该点的外法线方向的方向导数.

11. $\dfrac{6}{7}\sqrt{14}$

12. (1) $\dfrac{dz}{dx}=\dfrac{1}{1+x^2}$;

(2) $\dfrac{\partial z}{\partial u}=3u^2\sin v\cos v(\cos v-\sin v)$, $\dfrac{\partial z}{\partial v}=-2u^3\sin v\cos v(\sin v+\cos v)+u^3(\sin^3 v+\cos^3 v)$;

(3) $\dfrac{\partial z}{\partial u}=e^{uv}[v\sin(u+v)+\cos(u+v)]$, $\dfrac{\partial z}{\partial v}=e^{uv}[u\sin(u+v)\cos(u+v)]$;

(4) $\dfrac{\partial z}{\partial x}=2xf_1'+ye^{xy}f_2'$; $\dfrac{\partial z}{\partial y}=-2yf_1'+xe^{xy}f_2'$,

$$\dfrac{\partial^2 z}{\partial x^2}=2f_1'+y^2e^{xy}f_2'+4x^2f_{11}''+4xye^{xy}f_{12}''+y^2e^{2xy}f_{22}'',$$

$$\dfrac{\partial^2 z}{\partial x\partial y}=-4xyf_{11}''+2x^2e^{xy}f_{12}''+(1+xy)e^{xy}f_2'-2y^2e^{xy}f_{12}''+xye^{2xy}f_{22}'';$$

(5) $\dfrac{\partial z}{\partial x}=-\dfrac{F_1'+F_2'}{F_3'}-1$; $\dfrac{\partial z}{\partial y}=-\dfrac{F_2'}{F_3'}-1$.