习题 6.1

1. (1) $\{(x,y) \mid x^2+y^2 \leqslant 1,\ y > \sqrt{x} \geqslant 0\}$；(2) $x^4-2x^2y^2+2y^4$；(3) $2y+(x-y)^2$；
(4) $x+y \geqslant 1$；(5) $-1 \leqslant x \leqslant 1,\ y \neq 0$；(6) $k^2 \cdot f(x,y)$；(7) $\dfrac{x^2-y^2}{2x}$；
(8) π；(9) $\ln 2$；(10) $\{(x,y) \mid y^2-2x=0\}$；(11) $\left(\dfrac{\pi}{e}\right)^2$；(12) $f_x(0,1)=1$；
(13) $-y$；(14) $f_x(x,1)=1$；(15) $\dfrac{1}{y}$.

2. $(x+y)^{xy}+(xy)^{2x}$.

3. (1) 23；(2) $\dfrac{1}{x^3}-\dfrac{2}{xy}+\dfrac{3}{y^2}$；(3) $1-\dfrac{2y}{x}+\dfrac{3y^2}{x^2}$；(4) $-2x+6y+3h$.

4. $t^2 f(x,y)$.

5. (1) -8；(2) 1；(3) $-\dfrac{1}{4}$；(4) 2.

6. 略.

7. $\{(x,y) \mid y^2=x\}$.

8. (1) $\dfrac{\partial z}{\partial x}=-\dfrac{y}{x^2+y^2}$；$\dfrac{\partial z}{\partial y}=\dfrac{x}{x^2+y^2}$；(2) $\dfrac{\partial z}{\partial x}=3x^2y-y^3$；$\dfrac{\partial z}{\partial y}=x^3-3y^2x$；
(3) $\dfrac{\partial z}{\partial x}=y[\cos(xy)-\sin(2xy)]$；$\dfrac{\partial z}{\partial y}=x[\cos(xy)-\sin(2xy)]$；
(4) $\dfrac{\partial z}{\partial x}=\dfrac{1}{y}\cos\dfrac{x}{y}+\mathrm{e}^{-xy}-xy\mathrm{e}^{-xy}$；$\dfrac{\partial z}{\partial y}=-\dfrac{x}{y^2}\cos\dfrac{x}{y}-x^2\mathrm{e}^{-xy}$；
(5) $\dfrac{\partial u}{\partial x}=\dfrac{z}{y}\left(\dfrac{x}{y}\right)^{z-1}$；$\dfrac{\partial u}{\partial y}=-\dfrac{zx^z}{y^{z+1}}$；$\dfrac{\partial u}{\partial z}=\left(\dfrac{x}{y}\right)^z\ln\dfrac{x}{y}$；
(6) $\dfrac{\partial u}{\partial x}=-\dfrac{2xz}{(x^2+y^2)^2}$；$\dfrac{\partial u}{\partial y}=-\dfrac{2yz}{(x^2+y^2)^2}$；$\dfrac{\partial u}{\partial z}=\dfrac{1}{x^2+y^2}$.

9. (1) $\dfrac{\partial z}{\partial x}\quad=1$；$\dfrac{\partial z}{\partial y}\quad=1+2\ln 2$；(2) $\dfrac{\partial z}{\partial x}\quad=\dfrac{\pi}{4}\sin\dfrac{2}{\pi}$；$\dfrac{\partial z}{\partial y}\quad=-\dfrac{1}{2}\sin\dfrac{2}{\pi}$.

10. (1) $\dfrac{\partial^2 z}{\partial x^2}=\dfrac{2xy}{(x^2+y^2)^2}$，$\dfrac{\partial^2 z}{\partial y^2}=-\dfrac{2xy}{(x^2+y^2)^2}$，$\dfrac{\partial^2 z}{\partial x\partial y}=\dfrac{y^2-x^2}{(x^2+y^2)^2}$
(2) $\dfrac{\partial^2 z}{\partial x^2}=y^x\ln^2 y$，$\dfrac{\partial^2 z}{\partial y^2}=x(x-1)y^{x-2}$，$\dfrac{\partial^2 z}{\partial x\partial y}=y^{x-1}(1+x\ln y)$

11. 略.

12. $\dfrac{\pi}{6}$.

13. 提示：(0，0)处的偏导数应按定义求

$$f_x(x,y)=\begin{cases}\dfrac{2xy^3}{(x^2+y^2)^2} & x^2+y^2\neq 0\\ 0 & x^2+y^2=0\end{cases},\quad f_y(x,y)=\begin{cases}\dfrac{x^2(x^2-y^2)}{(x^2+y^2)^2} & x^2+y^2\neq 0\\ 0 & x^2+y^2=0\end{cases}$$

14. (1) $\Delta_x z=(2x-y)\Delta x+(\Delta x)^2$；$\Delta_y z=(1-x)\Delta y$；

$\Delta z=(2x-y)\Delta x+(1-x)\Delta y-\Delta x\Delta y+(\Delta x)^2$；

(2) $\Delta_x z=0.21$；$\Delta_y z=0.1$；$\Delta z=0.32$.

15. (1) $dz=(y+\frac{1}{y})dx+(x-\frac{x}{y^2})dy$；(2) $dz=\frac{1}{\sqrt{y^2-x^2}}dx-\frac{x}{y\sqrt{y^2-x^2}}dy$；

(3) $dz=\frac{x}{x^2+y^2}dx+\frac{y}{x^2+y^2}dy$；(4) $du=e^x[(x^2+y^2+z^2+2x)dx+2ydy+2zdz]$.

16. $df(1,1,1)=dx-dy$.

17. 0.502.

18. $dz=-0.2$，$\Delta z\approx -0.2040402$.

19. 解：(1) $f(x,y)$在$(0,0)$处连续；(2) $f'_x(0,0)=0$，$f'_y(0,0)=0$；(3) $f(x,y)$在$(0,0)$处可微分

20. $\frac{2}{5}$.　　21. $\frac{\sqrt{2}}{2}$.

22. $\frac{68}{13}$.　　23. $1+2\sqrt{3}$.

24. $\text{grad}f(0,0,0)=3\boldsymbol{i}-2\boldsymbol{j}-6\boldsymbol{k}$，$\text{grad}f(1,1,1)=6\boldsymbol{i}+3\boldsymbol{j}$

25. $\text{grad}u=2\boldsymbol{i}-4\boldsymbol{j}+\boldsymbol{k}$ 是方向导数取最大值的方向，此方向导数的最大值为 $|\text{grad}u|=\sqrt{21}$