

习题 6.2

$$1. \frac{dz}{dt} = -e^t - e^{-t}.$$

$$2. \frac{dz}{dt} = e^{\sin t - 2t^3} (\cos t - 6t^2).$$

$$3. \frac{dz}{dt} = \frac{3 - 12t^2}{\sqrt{1 - (3t - 4t^3)^2}}.$$

$$4. \frac{dz}{dt} = \frac{e^x(1+x)}{1+x^2e^{2x}}.$$

$$5. \frac{dz}{dt} = \left(3 - \frac{4}{t^3} - \frac{1}{2\sqrt{t}}\right) \sec^2\left(3t + \frac{2}{t^2} - \sqrt{t}\right).$$

$$6. \frac{du}{dx} = e^{ax} \sin x. \quad 7. -1. \quad 8. 2.$$

$$9. (1) \frac{dz}{dx} = \frac{1}{1+x^2};$$

$$(2) \frac{\partial z}{\partial u} = 3u^2 \sin v \cos v (\cos v - \sin v), \quad \frac{\partial z}{\partial v} = -2u^3 \sin v \cos v (\sin v + \cos v) + u^3 (\sin^3 v + \cos^3 v);$$

$$(3) \frac{\partial z}{\partial u} = e^{uv} [v \sin(u+v) + \cos(u+v)], \quad \frac{\partial z}{\partial v} = e^{uv} [u \sin(u+v) + \cos(u+v)];$$

$$(4) \frac{\partial z}{\partial x} = 2xf_1 + ye^{xy}f_2; \quad \frac{\partial z}{\partial y} = -2yf_1 + xe^{xy}f_2;$$

$$\frac{\partial^2 z}{\partial x^2} = 2f_1 + y^2 e^{xy}f_2 + 4x^2 f_{11} + 4xye^{xy}f_{12} + y^2 e^{2xy}f_{22};$$

$$\frac{\partial^2 z}{\partial x \partial y} = -4xyf_{11} + 2x^2 e^{xy}f_{12} + (1+xy)e^{xy}f_2 - 2y^2 e^{xy}f_{12} + xye^{2xy}f_{22};$$

$$(5) \frac{\partial z}{\partial x} = -\frac{F_1 + F_2}{F_3} - 1; \quad \frac{\partial z}{\partial y} = -\frac{F_2}{F_3} - 1.$$

$$10. \text{略.} \quad 11. \text{略.} \quad 12. \text{略.}$$

$$13. \text{提示: 将 } \xi, \eta \text{ 看作中间变量, 通过复合函数偏导数运算求得新方程为 } \frac{\partial^2 u}{\partial \xi \partial \eta} = 0$$

$$14. \frac{\partial^2 z}{\partial x^2} = 4x^2 f'' + 2f', \quad \frac{\partial^2 z}{\partial x \partial y} = 4xy f'', \quad \frac{\partial^2 z}{\partial y^2} = 4y^2 f'' + 2f''$$

$$15. \frac{\partial^2 z}{\partial x \partial y} = -2f'' + g_{12}x + g_2 + xyg_{22}$$

$$16. (1) \frac{\partial^2 z}{\partial x^2} = -a^2 \sin(ax+by), \quad \frac{\partial^2 z}{\partial x^2 \partial y} = -ab \sin(ax+by), \quad \frac{\partial^2 z}{\partial y^2} = -b^2 \sin(ax+by);$$

$$(2) \frac{\partial^2 z}{\partial x^2} = \frac{xy^3}{\sqrt{(1-x^2y^2)^3}}, \quad \frac{\partial^2 z}{\partial x \partial y} = \frac{1}{\sqrt{(1-x^2y^2)^3}}, \quad \frac{\partial^2 z}{\partial y^2} = \frac{x^3y}{\sqrt{(1-x^2y^2)^3}};$$

$$(3) \frac{\partial^2 x}{\partial x^2} = 2y(2y-1)x^{2y-2}, \quad \frac{\partial^2 z}{\partial x \partial y} = 2x^{(2y-1)}(1+2y \ln x), \quad \frac{\partial^2 z}{\partial y^2} = 4x^{2y} \ln^2 x;$$

$$(4) \frac{\partial^2 x}{\partial x^2} = \frac{\ln y (\ln y - 1)}{x^2} y^{\ln x}, \quad \frac{\partial^2 z}{\partial x \partial y} = \frac{\ln x \ln y + 1}{xy} y^{\ln x}, \quad \frac{\partial^2 z}{\partial y^2} = \frac{\ln x (\ln x - 1)}{y^2} y^{\ln x};$$

$$17. \text{ 略} \quad 18. 2, 4, 0.$$

$$19. \frac{\partial^2 u}{\partial y \partial x} = f_2 + x f_{21} + x y f_{22} + 2 x y z f_{23} + z f_3 + x z f_{31} + x y z^2 f_{33},$$

$$\frac{\partial^2 u}{\partial z \partial y} = x f_3 + x^2 y f_{32} + x^2 y z f_{33},$$

$$\frac{\partial^2 u}{\partial x \partial z} = y f_3 + x y f_{13} + x y^2 f_{23} + x y^2 z f_{33}.$$

$$20. \frac{\partial^2 u}{\partial x^2} = 4 x^2 f''(x^2 + y^2 + z^2) + 2 f'(x^2 + y^2 + z^2).$$

$$21. \text{ 略} \quad 22. \frac{dy}{dx} = -\frac{b^2 x}{a^2 y}. \quad 23. \frac{dy}{dx} = \frac{y[\cos(xy) - e^{xy} - 2x]}{x[x + e^{xy} - \cos(xy)]}.$$

$$24. \frac{dy}{dx} = \frac{x+y}{x-y}. \quad 25. \frac{\partial z}{\partial x} = \frac{yz - \sqrt{xyz}}{\sqrt{xyz} - xy}, \quad \frac{\partial z}{\partial y} = \frac{xz - 2\sqrt{xyz}}{\sqrt{xyz} - xy}.$$

$$26. \frac{\partial z}{\partial x} = \frac{yz}{e^z - xy}, \quad \frac{\partial z}{\partial y} = \frac{xz}{e^z - xy}.$$

$$27. \frac{\partial z}{\partial x} = \frac{azy - x}{z - axy}, \quad \frac{\partial z}{\partial y} = \frac{axz - y}{z - axy}.$$

$$28. \frac{\partial z}{\partial x} = \frac{z}{x+z}, \quad \frac{\partial z}{\partial y} = \frac{z^2}{y(x+z)}.$$

$$29. dz = -\frac{z}{x} dx - \frac{-2xyz^2 + z}{2xyz - 2xy^2z + y} dy.$$

$$30. \frac{dy}{dx} = -\frac{x(6z+1)}{2y(3z+1)}, \quad \frac{dz}{dx} = \frac{x}{3z+1}.$$

$$31. \frac{\partial u}{\partial x} = -\frac{xu + yv}{x^2 + y^2}, \quad \frac{\partial u}{\partial y} = \frac{xv - yu}{x^2 + y^2}, \quad \frac{\partial v}{\partial x} = -\frac{xv - yu}{x^2 + y^2}, \quad \frac{\partial v}{\partial y} = -\frac{xu + yv}{x^2 + y^2}.$$

$$32. \frac{\partial z}{\partial x} = \frac{4x\sqrt{z} - 2\sqrt{z}e^{(y-x)^2}}{2\sqrt{z} + e^z}, \quad \frac{\partial z}{\partial y} = \frac{2\sqrt{z}e^{(y-x)^2}}{2\sqrt{z} + e^z}.$$

$$33. \frac{\partial u}{\partial x} = \frac{-uf_1(2yvg_2 - 1) - f_2g_1}{(xf_1 - 1)(2yvg_2 - 1) - f_2g_1}, \quad \frac{\partial v}{\partial x} = \frac{(1 - xf_1)v^2g_2 + f_2g_1}{(1 - xf_1)(1 - 2yvg_2) - f_2g_1}$$

$$34. \frac{\partial^2 z}{\partial x^2} = \frac{2y^2ze^z - 2xy^3z - y^2z^2e^z}{(e^z - xy)^3}.$$

$$35. dz = -\frac{2xF_2}{yF_1} dx - \frac{z}{y} dy. \quad 36. \frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x} = -1.$$

$$37. \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1.$$

$$38. 1) \frac{dx}{dz} = \frac{y-z}{x-y}, \quad \frac{dy}{dz} = \frac{z-x}{x-y}. \quad 2) \frac{\partial u}{\partial x} = \frac{\sin v}{e^u(\sin v - \cos v) + 1}, \quad \frac{\partial u}{\partial y} = \frac{-\cos v}{e^u(\sin v - \cos v) + 1}.$$

$$\frac{\partial v}{\partial y} = \frac{\cos v - e^u}{u[e^u(\sin v - \cos v) + 1]}, \quad \frac{\partial v}{\partial x} = \frac{\sin v + e^u}{u[e^u(\sin v - \cos v) + 1]}.$$