

习题 8.2

$$1. (1) y = Ce^{\sqrt{(1-x^2)}}; (2) y = C \sin x - 1;$$

$$(3) \text{ 当 } \sin \frac{y}{2} \neq 0 \text{ 时, 通解为 } \ln \left| \tan \frac{y}{4} \right| = C - 2 \sin \frac{x}{2}; \text{ 当 } \sin \frac{y}{2} = 0 \text{ 时, 特解为 } y = k\pi \\ (k=0, \pm 1, \pm 2, \cdots);$$

$$(4) \sin \frac{y}{x} = \ln |x| + C; (5) xy = Ce^{-\arctan \frac{y}{x}};$$

$$(6) \ln[(x-1)^2 + 4y^2] + \arctan \frac{2y}{x-1} = C;$$

$$(7) x + 3y + 2 \ln |x + y - 2| = C (C = 2C_1);$$

$$(8) y = x^3 + Cx;$$

$$(9) x = Cy^3 + \frac{y^2}{2};$$

$$(10) x = \frac{Ce^{-y}}{y} + \frac{e^y}{2y};$$

$$(11) x = Ce^{\sin y} - 2(\sin y + 1);$$

$$(12) x = y^2 + Cy^2 e^{1/y};$$

$$(13) y = f(x) - 1 + Ce^{-f(x)};$$

$$(14) xy^{-3} + \frac{3}{4}x^2(2\ln x - 1) = C;$$

$$(15) y^{-\frac{1}{3}} = Cx^{\frac{2}{3}} - \frac{1}{7}x^3 \text{ 或 } 7y^{-\frac{1}{3}} = Cx^{\frac{2}{3}} - x^3;$$

$$(16) y = x + \frac{1}{z} = x + \frac{1}{Ce^{x^2/2} - x^2 - 2};$$

$$(17) (1+x^2)(1+y^2) = Cx^2;$$

$$(18) \arcsin \frac{y}{x} = \ln |x| + C; y^2 = x^2;$$

$$(19) y^2(x^2 + 1 + Ce^{x^2}) = 1, y = 0;$$

$$(20) x^3 + 3x^2y = C;$$

$$(21) x \sin(x+y) = C;$$

$$(22) \arctan x + xy = C \text{ (提示: 积分因子 } \frac{1}{1+x^2});$$

$$(23) \frac{x}{y} = \ln(x^2 + y^2) + C \text{ (提示: 方程两边同乘以 } \frac{1}{(x^2 + y^2)y^2}).$$

$$2. (1) \frac{y^2}{2} + \frac{y^3}{3} = \frac{x^2}{2} + \frac{x^3}{3};$$

$$(2) y = \frac{2}{3}(4 - e^{-3x});$$

$$(3) \text{ 通解 } y = \frac{1}{C + \ln|1+x|}, \text{ 另有解 } y = 0, \text{ 特解 } y = \frac{1}{1 + \ln|1+x|};$$

(4) 通解为 $x = \frac{1}{2} \sin y + C \csc y$, 特解为 $x = \frac{1}{2} \sin y + \frac{3}{8} \csc y$.

3. $y(x) = e^x(x+1)$.

4. (1) 令 $x+y=u$, 通解为 $\csc(x+y) - \cot(x+y) = \frac{C}{x}$;

(2) 令 $xy=u$, 通解为 $x = Cy e^{\frac{1}{xy}}$;

(3) 令 $u=y^2$, 通解为 $y^2 = e^{-\frac{1}{2}x^2} \left(\int e^{\frac{1}{2}x^2} \sin x dx + C \right)$;

(4) 令 $z = \sin y$, $u = z^{-1}$, 通解为 $\frac{2}{\sin y} + \cos x + \sin x = C e^{-x}$, 另加特解 $y = k\pi$;

(5) 令 $\frac{y^3}{x} = u$, 通解为 $(y^3 - 3x)^7 (y^3 + 2x)^3 = C x^{15}$;

(6) 令 $u = xy$, 通解为 $y = -\frac{1}{2x} \tan(x^2 + C)$;

(7) 令 $y = \frac{y^2}{x}$, 通解为 $\sin \frac{y^2}{x} = Cx$;

(8) 令 $u = e^y$, 通解为 $y = -\ln(Cx - x^2)$.

5. 约为 1.5 g/L.

6. $\varphi(t) = e^{t\varphi'(0)}$.

7. 提示: 将 $y = e^x$ 代入原微分方程, 可求出 $P(x) = x(e^{-x} - 1)$, 可求出通解为 $y = C e^{x+e^{-x}} + e^x$, 又初始条件可求得特解为 $y = e^x - e^{x+e^{-x}-\frac{1}{2}}$.