习题3

1. 选择题

(1)D; (2)B; (3)C; (4)C; (5)D; (6)A; (7)D; (8)C; (9)B; (10)D.

2. 填空题

(1) $\frac{1}{6}f(3x^2+1)+C$; (2) $f(1)=e^{-1}$; (3) $\frac{x^3}{3}+\frac{1}{3}(x^2-1)^{\frac{3}{2}}+C$;

(4) $\frac{1}{2}\arctan x^2+\frac{1}{4}\ln(1+x^4)+C$; (5) $-\frac{4}{3}\sqrt{1-x\sqrt{x}}+C$; (6) $\frac{1}{12}$;

(7) $f(x)=x^2+\frac{1}{3}e^{1-x}$; (8) $2\ln(1+\pi)$; (9) $\sqrt{2}x+y=\frac{\sqrt{2\pi}}{2}$; (10) $2\sqrt{2}$.

3. 计算下列不定积分:

(1) $\frac{1}{2\sqrt{2}}\ln\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right|+C$; (2) $\frac{1}{2}(\arcsin x)^2-\frac{\sqrt{1-x^2}}{x}\arcsin x+\ln|x|+C$;

(3) $-\frac{\ln x}{1+x}+\ln\left|\frac{x}{1+x}\right|+C$; (4) $\frac{1}{5}\ln|\sec x+\tan x|-\frac{3}{10}\arctan(\frac{1}{2}\tan\frac{x}{2})+C$;

(5) $x^2-x+\frac{1}{3}\ln|x-1|-\frac{1}{6}\ln(x^2+x+1)+\frac{\sqrt{3}}{3}\arctan\frac{2x+1}{\sqrt{3}}+C$;

(6) $\frac{x}{2}+\sqrt{x}-\frac{1}{2}\sqrt{x(1+x)}-\frac{1}{2}\ln(\sqrt{x}+\sqrt{1+x})+C$;

(7) $\int(\sin x+\cos x)^n\cos 2x\,dx=\begin{cases}\ln|\sin x+\cos x|+C, & n=-2,\\ \frac{1}{n+2}(\sin x+\cos x)^{n+2}+C, & n\neq -2.\end{cases}$

(8) $\frac{1}{a^2+b^2}(ax-b\ln|a\sin x+b\cos x|)+C$.

4. $f(x)=\frac{1}{2}\tan(x-\frac{\pi}{4})+\frac{1}{2}=\frac{\tan x}{1+\tan x}$.

5. $f(x)=\begin{cases}\frac{1}{3}x^3+C, & x\geqslant 0,\\ 1-\cos x+C, & x<0.\end{cases}$

6. $f(x)=\frac{xe^{\frac{x}{2}}}{2(1+x)^{\frac{3}{2}}}$.

7. (1) $-\frac{1}{6}$; (2) $\pi(e-1)$; (3) $\begin{cases}\frac{x^2}{2}+x+\frac{1}{2}, & -1\leqslant x\leqslant 0,\\ -\frac{x^2}{2}+x+\frac{1}{2}, & x>0.\end{cases}$;

(4) $\frac{\pi}{4}+\frac{1}{2}\ln 2$; (5) $2(1-e^{-1})$

8. $\frac{\pi^2}{4}$. 9. $f(0)$. 10. 2. 11. 证明略. 12. 证明略. 13. 证明略.