习题 4.6

1. 正弦级数：$x+1=\frac{2}{\pi}\left[(\pi+2)\sin x-\frac{\pi}{2}\sin 2x+\frac{1}{3}(\pi+2)\sin 3x-\frac{\pi}{4}\sin 4x+\cdots\right]$，$(0\leqslant x\leqslant\pi)$

或余弦级数：$x+1=\frac{\pi+2}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2}$，$(0\leqslant x\leqslant\pi)$

2. $f(x)=\cos\frac{x}{2}=\frac{2}{\pi}+\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\pi}\frac{1}{n^2-\frac{1}{4}}\cos nx.$

3. $M(x)=\frac{2pl}{\pi^2}\left(\sin\frac{\pi x}{l}-\frac{1}{3^2}\sin\frac{3\pi x}{l}+\frac{1}{5^2}\sin\frac{5\pi x}{l}-\cdots\right)(0\leqslant x\leqslant l)$.

4. 取 $x=\frac{\pi}{3}$ 即得要证等式.

5. 1；

6. $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx=1-\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx,0\leqslant x\leqslant\pi,\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}=\frac{\pi^2}{12}.$

7. $\sin\frac{x}{2}=\frac{2}{\pi}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\cos nx$，$x\in[0,\pi]$；$\frac{1}{2}$.