

习题 5

1. (1) $(x - x_0, y - y_0, z - z_0)$, (x_0, y_0, z_0) ; (2) 线性相关; (3) 3.

2. $(0, 2, 0)$.

3. $\sqrt{30}$.

$$4. \overrightarrow{AD} = \frac{1}{2}\mathbf{a} + \mathbf{c}, \overrightarrow{BE} = \mathbf{a} + \frac{1}{2}\mathbf{b}, \overrightarrow{CF} = \mathbf{b} + \frac{1}{2}\mathbf{c},$$

证明: 因为 $\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB} = \mathbf{0}$,

$$\text{所以 } \mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}, \overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \frac{1}{2}(\mathbf{a} + \mathbf{b} + \mathbf{c}).$$

5. 证明: 设 $\triangle ABC$ 三边 $\overrightarrow{BC} = \mathbf{a}$, $\overrightarrow{CA} = \mathbf{b}$, $\overrightarrow{AB} = \mathbf{c}$ 三边中点依次为 D, E, F , $\overrightarrow{EF} = \overrightarrow{EA} + \overrightarrow{AF} = \frac{1}{2}(\mathbf{b} + \mathbf{c}) = -\frac{1}{2}\mathbf{a} = -\frac{1}{2}\overrightarrow{BC}$, 所以 $\overrightarrow{EF} \parallel \overrightarrow{BC}$ 且 $|\overrightarrow{EF}| = \frac{1}{2}|\overrightarrow{BC}|$ 即 $EF \parallel BC$ 且 $EF = \frac{1}{2}BC$ 结论成立.

$$6. 1. \quad 7. \arccos \frac{2}{\sqrt{7}}. \quad 8. \frac{\pi}{3}. \quad 9. -4, \frac{\pi}{4}.$$

10. 30.

11. $(14, 10, 2)$.

12. 证明: 令 $k_1\mathbf{a} + k_1\mathbf{b} + k_1\mathbf{c} = \mathbf{0}$, 联立方程组求得一组解: $(5, 1, -1)$, 所以 $5\mathbf{a} + \mathbf{b} - \mathbf{c} = \mathbf{0}$, $\mathbf{c} = 5\mathbf{a} + \mathbf{b}$.

$$13. x^2 + y^2 - 2x + 2y - 4z + 6 = 0.$$

14. (1) 母线: $\Gamma \begin{cases} z = y^2 \\ x = 0 \end{cases}$ 旋转轴 z 轴.

(2) 母线: $\Gamma \begin{cases} z = 3y^2 \\ x = 0 \end{cases}$ 旋转轴 z 轴, 母线: $\Gamma \begin{cases} x^2 - \frac{y^2}{4} = 1 \\ x = 0 \end{cases}$ 旋转轴 x 轴.

$$15. x \pm \sqrt{26}y + 3z - 3 = 0.$$

$$16. x + 2y + 1 = 0.$$

$$17. \frac{x+1}{4} = \frac{y}{19} = \frac{z-4}{7}.$$

$$18. \left(0, 0, \frac{1}{5}\right), \text{最小值为 } \frac{\sqrt{6}}{5}.$$

$$19. xOy \text{ 面: } \begin{cases} x^2 + y^2 - x - y = 0 \\ z = 0 \end{cases}; yOz \text{ 面: } \begin{cases} z^2 + 2y^2 + yz - 4y - 3z + 2 = 0 \\ x = 0 \end{cases};$$

$$zOx \text{ 面: } \begin{cases} 2x^2 + z^2 + 2xz - 3z - 4x + 2 = 0 \\ y = 0 \end{cases}.$$

$$20. xOy \text{ 面: } \begin{cases} (x-1)^2 + y^2 \leq 1 \\ z = 0 \end{cases}; yOz \text{ 面: } \begin{cases} (z^2 - 2)^2 + 4y^2 \leq 4 \\ x = 0 \end{cases}; zOx \text{ 面: } \begin{cases} x \leq z \leq \sqrt{2x} \\ y = 0 \end{cases}.$$

21. 略.