习题 6

1. (1) $D=\{(x,y)\mid x^2+2y^2\neq 0\}$；

(2) $D=\{(x,y)\mid x-2y+1>0\}$；

(3) $D=\{(x,y)\mid \frac{x^2}{a^2}+\frac{y^2}{b^2}\leqslant 1\}$；

(4) $D=\{(x,y)\mid x>y,\ x>-1\}$；

(5) $D=\{(x,y)\mid y\geqslant 0,\ x\geqslant 0,\ x^2\geqslant y\}$；

(6) $D=\{(x,y)\mid x>0,\ -x\leqslant y\leqslant x\}\cup\{(x,y)\mid x<0,\ x\leqslant y\leqslant -x\}$；

(7) $\{(x,y,z)\mid x>0,\ y>0,\ z>0\}$.

2. (1)1；(2)0；(3)$-1/4$；(4)1；(5)6.

3. (1) $\frac{\partial z}{\partial x}=\frac{2}{y\sin\frac{2x}{y}},\ \frac{\partial z}{\partial y}=\frac{-2x}{y^2\sin\frac{2x}{y}}$；

(2) $\frac{\partial z}{\partial x}=\frac{y}{2\sqrt{x(1-xy^2)}},\ \frac{\partial z}{\partial y}=\sqrt{\frac{x}{1-xy^2}}$；

(3) $\frac{\partial z}{\partial x}=\frac{y}{x^2}\sin\frac{x}{y}\sin\frac{y}{x}+\frac{1}{y}\cos\frac{y}{x}\cos\frac{x}{y},\ \frac{\partial z}{\partial y}=-\frac{x}{y^2}\cos\frac{x}{y}\cos\frac{y}{x}-\frac{1}{x}\sin\frac{x}{y}\sin\frac{y}{x}$；

(4) $\frac{\partial z}{\partial x}=-\frac{y}{x^2}3^{\frac{y}{x}}\ln 3,\ \frac{\partial z}{\partial y}=\frac{1}{x}3^{\frac{y}{x}}\ln 3$；

(5) $\frac{\partial z}{\partial x}=y\mathrm{e}^{\sin\pi xy}(1+\pi xy\cos\pi xy),\ \frac{\partial z}{\partial y}=x\mathrm{e}^{\sin\pi xy}(1+\pi xy\cos\pi xy)$；

(6) $\frac{\partial z}{\partial x}=\frac{1}{x+\ln y},\ \frac{\partial z}{\partial y}=\frac{1}{y(x+\ln y)}$；

(7) $\frac{\partial z}{\partial x}=\frac{1}{2\sqrt{x}}\sin\frac{y}{x}-\frac{y}{x\sqrt{x}}\cos\frac{y}{x},\ \frac{\partial z}{\partial y}=\frac{1}{\sqrt{x}}\cos\frac{y}{x}$；

(8) $\frac{\partial u}{\partial t}=\rho\varphi\mathrm{e}^{t\varphi}+1,\ \frac{\partial u}{\partial \rho}=\mathrm{e}^{t\varphi},\ \frac{\partial u}{\partial \varphi}=\rho t\mathrm{e}^{t\varphi}-\mathrm{e}^{-\varphi}$；

(9) $\frac{\partial u}{\partial \varphi}=\mathrm{e}^{\varphi+\theta}[\cos(\theta-\varphi)+\sin(\theta-\varphi)],\ \frac{\partial u}{\partial \theta}=\mathrm{e}^{\varphi+\theta}[\cos(\theta-\varphi)-\sin(\theta-\varphi)]$.

4. $f_x(x,y)=\begin{cases}\frac{2xy^3}{(x^2+y^2)^2} & x^2+y^2\neq 0\\ 0 & x^2+y^2=0\end{cases},\ f_y(x,y)=\begin{cases}\frac{x^2(x^2-y^2)}{(x^2+y^2)^2} & x^2+y^2\neq 0\\ 0 & x^2+y^2=0\end{cases}$

5. 略

6. (1) $\frac{\partial^2 z}{\partial x^2}=\frac{2}{y}\sec^2\frac{x^2}{y}+\frac{8x^2}{y^2}\sec^3\frac{x^2}{y}\sin\frac{x^2}{y},\ \frac{\partial^2 z}{\partial y^2}=\frac{2x^2}{y^3}\sec^2\frac{x^2}{y}+\frac{2x^4}{y^4}\sec^3\frac{x^2}{y}\sin\frac{x^2}{y}$,

$\frac{\partial^2 z}{\partial x\partial y}=-\frac{2x}{y^2}\sec^2\frac{x^2}{y}-\frac{4x^3}{y^3}\sec^3\frac{x^2}{y}\sin\frac{x^2}{y}$；

(2) $\frac{\partial^2 z}{\partial x^2}=-\frac{3xy^2}{(x^2+y^2)^{\frac{5}{2}}},\ \frac{\partial^2 z}{\partial y^2}=\frac{x(2y^2-x^2)}{(x^2+y^2)^{\frac{5}{2}}}$.

(3) $\frac{\partial^2 z}{\partial x^2}=\frac{xy^3}{\sqrt{(1-x^2y^2)^3}}$, $\frac{\partial^2 z}{\partial x\partial y}=\frac{1}{\sqrt{(1-x^2y^2)^3}}$.

(4) $\frac{\partial^2 y}{\partial y^2}=\frac{\ln x(\ln x-1)}{y^2}y^{\ln x}$, $\frac{\partial^2 z}{\partial x\partial y}=\frac{(\ln x\ln y+1)}{xy}y^{\ln x}$;

(5) $\frac{\partial^3 u}{\partial x\partial y\partial z}=(1+3xyz+x^2y^2z^2)e^{xyz}$.

7. (1) $dz=\left(ye^{xy}+\frac{1}{x+y}\right)dx+\left(xe^{xy}+\frac{1}{x+y}\right)dy$;

(2) $dz=\frac{dx}{1+x^2}+\frac{dy}{1+y^2}$;

(3) $dz=(xdy+ydx)\cos(xy)$;

(4) $dz=\frac{4xy(xdy-ydx)}{(x^2-y^2)^2}$;

(5) $dz=(2e^{-y}-\frac{\sqrt{3}}{2\sqrt{x}})dx-2xe^{-y}dy$;

(6) $du=e^{x(x^2+y^2+z^2)}[(3x^2+y^2+z^2)dx+2xydy+2xzdz]$;

(7) $du=x^{xy}[y(1+\ln x)dx+x\ln xdy]$;

(8) $du=\frac{3dx-2dy+dz}{3x-2y+z}$;

(9) $du=\frac{2(x-y)(dx-dy)}{1+(x-y)^4}$.

8. (1) $dz=-4(dx+dy)$; (2) $dz=2dx-dy$.

9. $\Delta z=0.02$, $dy=0.03$

10. $z=xy^2+x^2+3y$.

11. 0.005.

12. -30π cm^3.

13. $-\frac{9\sqrt{3}}{2}$;

14. $\frac{68}{13}$.

15. $\left.\frac{\partial u}{\partial l}\right|_{(x_0,y_0,z_0)}=x_0+y_0+z_0$ 处沿球面在该点的外法线方向的方向导数.

16. $1+2\sqrt{3}$.

17. $\frac{6}{7}\sqrt{14}$

18. $\mathrm{grad}f(0,0,0)=3\boldsymbol{i}-2\boldsymbol{j}-6\boldsymbol{k}$, $\mathrm{grad}f(1,1,1)=6\boldsymbol{i}+3\boldsymbol{j}$

19. (1) $\frac{dz}{dx}=\frac{1}{1+x^2}$;

(2) $\frac{\partial z}{\partial u}=3u^2\sin v\cos v(\cos v-\sin v)$, $\frac{\partial z}{\partial v}=-2u^3\sin v\cos v(\sin v+\cos v)+u^3(\sin^3 v+\cos^3 v)$;

(3) $\frac{\partial z}{\partial u}=\mathrm{e}^{uv}[v\sin(u+v)+\cos(u+v)]$, $\frac{\partial z}{\partial v}=\mathrm{e}^{uv}[u\sin(u+v)\cos(u+v)]$;

(4) $\frac{\partial z}{\partial x}=2xf_1'+y\mathrm{e}^{xy}f_2'$; $\frac{\partial z}{\partial y}=-2yf_1'+x\mathrm{e}^{xy}f_2'$,

$\frac{\partial^2 z}{\partial x^2}=2f_1'+y^2\mathrm{e}^{xy}f_2'+4x^2f_{11}''+4xy\mathrm{e}^{xy}f_{12}''+y^2\mathrm{e}^{2xy}f_{22}''$,

$\frac{\partial^2 z}{\partial x\partial y}=-4xyf_{11}''+2x^2\mathrm{e}^{xy}f_{12}''+(1+xy)\mathrm{e}^{xy}f_2'-2y^2\mathrm{e}^{xy}f_{12}''+xy\mathrm{e}^{2xy}f_{22}''$;

(5) $\frac{\partial z}{\partial x}=-\frac{F_1'+F_2'}{F_3'}-1$; $\frac{\partial z}{\partial y}=-\frac{F_2'}{F_3'}-1$.

20. 略

21. 略

22. 1) $\frac{\mathrm{d}x}{\mathrm{d}z}=\frac{y-z}{x-y}$, $\frac{\mathrm{d}y}{\mathrm{d}z}=\frac{z-x}{x-y}$

2) $\frac{\partial u}{\partial x}=\frac{\sin v}{\mathrm{e}^u(\sin v-\cos v)+1}$, $\frac{\partial u}{\partial y}=\frac{-\cos v}{\mathrm{e}^u(\sin v-\cos v)+1}$

$\frac{\partial v}{\partial x}=\frac{\cos v-\mathrm{e}^u}{u[\mathrm{e}^u(\sin v-\cos v)+1]}$, $\frac{\partial v}{\partial y}=\frac{\sin v+\mathrm{e}^u}{u[\mathrm{e}^u(\sin v-\cos v)+1]}$

23. 切线方程：$\frac{x-\frac{1}{2}}{0}=\frac{y-2}{-1}=\frac{z-1}{2}$，法平面方程：$2z-y=0$.

24. 切线方程：$\frac{\sqrt{2}x-a}{-a}=\frac{\sqrt{2}y-a}{a}=\frac{4z-b\pi}{4b}$，法平面方程：$2\sqrt{2}a(x-y)-b(4z-b\pi)=0$

25. 切平面方程：$x+2y-z+5=0$，法线方程：$\frac{x-2}{-1}=\frac{y+3}{-2}=\frac{z-1}{1}$.

26. 切平面方程：$2ax_0x+2by_0y-z-z_0=0$，法线方程：$\frac{x-x_0}{2ax_0}=\frac{y-y_0}{2by_0}=\frac{z-z_0}{-1}$.

27. $\arccos\frac{3}{\sqrt{22}}$.