

习题 6.1

- $\{(x, y) \mid x^2 + y^2 \leq 1, y > \sqrt{x} \geq 0\}$; (2) $x^4 - 2x^2y^2 + 2y^4$; (3) $2y + (x - y)^2$;
 - $x + y \geq 1$; (5) $-1 \leq x \leq 1, y \neq 0$; (6) $k^2 \cdot f(x, y)$; (7) $\frac{x^2 - y^2}{2x}$;
 - π ; (9) $\ln 2$; (10) $\{(x, y) \mid y^2 - 2x = 0\}$; (11) $\left(\frac{\pi}{e}\right)^2$; (12) $f_x(0, 1) = 1$;
 - $-y$; (14) $f_x(x, 1) = 1$; (15) $\frac{1}{y}$.
 - $(x + y)^{xy} + (xy)^{2x}$.
 - 23; (2) $\frac{1}{x^3} - \frac{2}{xy} + \frac{3}{y^2}$; (3) $1 - \frac{2y}{x} + \frac{3y^2}{x^2}$; (4) $-2x + 6y + 3h$.
 - $t^2 f(x, y)$.
 - 8; (2) 1; (3) $-\frac{1}{4}$; (4) 2.
 - 略.
 - $\{(x, y) \mid y^2 = x\}$.
 - $\frac{\partial z}{\partial x} = -\frac{y}{x^2 + y^2}$; $\frac{\partial z}{\partial y} = \frac{x}{x^2 + y^2}$; (2) $\frac{\partial z}{\partial x} = 3x^2y - y^3$; $\frac{\partial z}{\partial y} = x^3 - 3y^2x$;
 - $\frac{\partial z}{\partial x} = y[\cos(xy) - \sin(2xy)]$; $\frac{\partial z}{\partial y} = x[\cos(xy) - \sin(2xy)]$;
 - $\frac{\partial z}{\partial x} = \frac{1}{y} \cos \frac{x}{y} + e^{-xy} - xye^{-xy}$; $\frac{\partial z}{\partial y} = -\frac{x}{y^2} \cos \frac{x}{y} - x^2 e^{-xy}$;
 - $\frac{\partial u}{\partial x} = \frac{z}{y} \left(\frac{x}{y}\right)^{z-1}$; $\frac{\partial u}{\partial y} = -\frac{zx^z}{y^{z+1}}$; $\frac{\partial u}{\partial z} = \left(\frac{x}{y}\right)^z \ln \frac{x}{y}$;
 - $\frac{\partial u}{\partial x} = -\frac{2xz}{(x^2 + y^2)^2}$; $\frac{\partial u}{\partial y} = -\frac{2yz}{(x^2 + y^2)^2}$; $\frac{\partial u}{\partial z} = \frac{1}{x^2 + y^2}$.
 - $\frac{\partial z}{\partial x} = 1$; $\frac{\partial z}{\partial y} = 1 + 2\ln 2$; (2) $\frac{\partial z}{\partial x} = \frac{\pi}{4} \sin \frac{2}{\pi}$; $\frac{\partial z}{\partial y} = -\frac{1}{2} \sin \frac{2}{\pi}$.
 - $\frac{\partial^2 z}{\partial x^2} = \frac{2xy}{(x^2 + y^2)^2}$, $\frac{\partial^2 z}{\partial y^2} = -\frac{2xy}{(x^2 + y^2)^2}$, $\frac{\partial^2 z}{\partial x \partial y} = \frac{y^2 - x^2}{(x^2 + y^2)^2}$
 - $\frac{\partial^2 z}{\partial x^2} = y^x \ln^2 y$, $\frac{\partial^2 z}{\partial y^2} = x(x-1)y^{x-2}$, $\frac{\partial^2 z}{\partial x \partial y} = y^{x-1}(1 + x \ln y)$
 - 略.
 - $\frac{\pi}{6}$.
 - 提示: $(0, 0)$ 处的偏导数应按定义求
- $$f_x(x, y) = \begin{cases} \frac{2xy^3}{(x^2 + y^2)^2} & x^2 + y^2 \neq 0 \\ 0 & x^2 + y^2 = 0 \end{cases}, f_y(x, y) = \begin{cases} \frac{x^2(x^2 - y^2)}{(x^2 + y^2)^2} & x^2 + y^2 \neq 0 \\ 0 & x^2 + y^2 = 0 \end{cases}$$
- $(1) \Delta_x z = (2x - y) \Delta x + (\Delta x)^2$; $\Delta_y z = (1 - x) \Delta y$;

$$\Delta z = (2x - y)\Delta x + (1 - x)\Delta y - \Delta x\Delta y + (\Delta x)^2;$$

$$(2) \Delta_x z = 0.2; \Delta_y z = 0.1; \Delta z = 0.32.$$

$$15. (1) dz = (y + \frac{1}{y})dx + (x - \frac{x}{y^2})dy; (2) dz = \frac{1}{\sqrt{y^2 - x^2}}dx - \frac{x}{y\sqrt{y^2 - x^2}}dy;$$

$$(3) dz = \frac{x}{x^2 + y^2}dx + \frac{y}{x^2 + y^2}dy; (4) du = e^x[(x^2 + y^2 + z^2 + 2x)dx + 2ydy + 2zdz].$$

$$16. df(1, 1, 1) = dx - dy.$$

$$17. 0.502.$$

$$18. dz = -0.2, \Delta z \approx -0.20404.$$

19. 解: (1) $f(x, y)$ 在 $(0, 0)$ 处连续; (2) $f'_x(0, 0) = 0, f'_y(0, 0) = 0$; (3) $f(x, y)$ 在 $(0, 0)$ 处可微分

$$20. \frac{2}{5}. \quad 21. \frac{\sqrt{2}}{2}.$$

$$22. \frac{68}{13}. \quad 23. 1 + 2\sqrt{3}.$$

$$24. \operatorname{grad} f(0, 0, 0) = 3\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}, \operatorname{grad} f(1, 1, 1) = 6\mathbf{i} + 3\mathbf{j}$$

$$25. \operatorname{grad} u = 2\mathbf{i} - 4\mathbf{j} + \mathbf{k} \text{ 是方向导数取最大值的方向, 此方向导数的最大值为 } |\operatorname{grad} u| = \sqrt{21}$$