

习题 1

$$1. (1) \begin{pmatrix} 35 \\ 6 \\ 49 \end{pmatrix}; (2) 14; (3) \begin{pmatrix} 7 & 14 & 21 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix};$$

$$(4) a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3.$$

$$2. \mathbf{AB} = \begin{pmatrix} 12 & 10 & 2 \\ 0 & 2 & 1 \\ 8 & 10 & 3 \end{pmatrix}, (\mathbf{AB})\mathbf{C} = \begin{pmatrix} 62 & 2 & 18 \\ 7 & -2 & -3 \\ 53 & -2 & 7 \end{pmatrix} = \mathbf{A}(\mathbf{BC}), \mathbf{BC} = \begin{pmatrix} 15 & -1 & 1 \\ 8 & 1 & 4 \end{pmatrix}.$$

$$3. f(\mathbf{A}) = \begin{pmatrix} 21 & -18 & 20 \\ -8 & 34 & 15 \\ -4 & 27 & 25 \end{pmatrix}.$$

$$5. (1) \text{取 } \mathbf{A} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \neq \mathbf{O}, \text{ 而 } \mathbf{A}^2 = \mathbf{O}; (2) \text{取 } \mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \text{ 有 } \mathbf{A} \neq \mathbf{O}, \mathbf{A} \neq \mathbf{E} \text{ 而 } \mathbf{A}^2 = \mathbf{A}; (3) \text{取 } \mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{X} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{Y} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \text{ 有 } \mathbf{X} \neq \mathbf{Y} \text{ 而 } \mathbf{AX} = \mathbf{AY}.$$

$$6. (1) 10^{25}\mathbf{E}, 10^{25}\begin{pmatrix} 3 & 1 \\ 1 & -3 \end{pmatrix}; (2) -8^{99}\begin{pmatrix} 2 & 4 & 8 \\ 1 & 2 & 4 \\ -3 & -6 & -12 \end{pmatrix}.$$

$$7. (1) \mathbf{A}^2 = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}, \mathbf{A}^3 = \begin{pmatrix} 1 & 0 \\ 3\lambda & 1 \end{pmatrix}, \dots, \mathbf{A}^k = \begin{pmatrix} 1 & 0 \\ k\lambda & 1 \end{pmatrix}; (2) \begin{pmatrix} \lambda^4 & 4\lambda^3 & 6\lambda^2 \\ 0 & \lambda^4 & 4\lambda^3 \\ 0 & 0 & \lambda^4 \end{pmatrix}.$$

$$8. (1) 0; (2) 8; (3) \frac{n(n-1)}{2}; (4) n(n-1).$$

$$9. -a_{11}a_{23}a_{32}a_{44}, a_{11}a_{23}a_{34}a_{42}.$$

$$10. (1) -4; (2) (a-b)(b-c)(c-a); (3) 0; (4) abcd + ab + cd + ad + 1; (5) a_3x^3 + a_2x^2 + a_1x + a_0; (6) 0.$$

$$11. (1) -3 \text{ 或 } \pm\sqrt{3}; (2) a, b, c.$$

$$12. (1) 1 + a_1 + \dots + a_n; (2) a_1a_2\dots a_n\left(1 + \sum_{i=1}^n \frac{1}{a_i}\right); (3) \prod_{n+1 \geq i > j \geq 1} (i-j);$$

$$(4) \left(1 + \sum_{i=1}^n \frac{a_i}{x_i - a_i}\right) \prod_{i=1}^n (x_i - a_i).$$

$$13. 24.$$

$$14. n! \left(1 - \sum_{j=2}^n \frac{1}{j}\right).$$

$$16. \mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - \mathbf{E}), (\mathbf{A} + 2\mathbf{E})^{-1} = \frac{1}{4}(3\mathbf{E} - \mathbf{A}).$$

$$17. -16. \quad 18. \mathbf{X} = \frac{1}{3} \begin{pmatrix} 23 & -7 & -13 \\ -22 & 5 & 14 \end{pmatrix}. \quad 19. \begin{pmatrix} 0 & 3 & 3 \\ -1 & 2 & 3 \\ 1 & 1 & 0 \end{pmatrix}.$$

$$20. \mathbf{B} = \mathbf{A} + \mathbf{E} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}. \quad 21. \mathbf{B} = \text{diag}\{6, 6, 6, -1\}.$$

$$22. \mathbf{B} = 2\mathbf{A} = 2\text{diag}\{1, -2, 1\}. \quad 23. 4 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad 27. \begin{pmatrix} \mathbf{0} & \mathbf{B}^{-1} \\ \mathbf{A}^{-1} & \mathbf{0} \end{pmatrix}.$$

$$28. (1) \begin{pmatrix} 1 & -2 & 0 & 0 \\ -2 & 5 & 0 & 0 \\ 0 & 0 & 2 & -3 \\ 0 & 0 & -5 & 8 \end{pmatrix}; (2) \frac{1}{2} \begin{pmatrix} 0 & 3 & -1 \\ 0 & -4 & 2 \\ 10 & 0 & 0 \end{pmatrix}.$$

$$29. |\mathbf{A}^8| = 10^{16}, \mathbf{A}^4 = \begin{pmatrix} 5^4 & 0 & 0 & 0 \\ 0 & 5^4 & 0 & 0 \\ 0 & 0 & 2^4 & 0 \\ 0 & 0 & 2^6 & 2^4 \end{pmatrix}.$$

$$30. a \neq 0, b \neq \frac{3}{2}.$$