

习题2

$$1. (1) \frac{1}{5} \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}; (2) \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}; (3) \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix};$$

$$(4) \begin{pmatrix} 0 & \frac{1}{a_1} & 0 & \cdots & 0 \\ 0 & 0 & \frac{1}{a_2} & \cdots & 1 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{a_{n-1}} \\ \frac{1}{a_n} & 0 & 0 & \cdots & 0 \end{pmatrix}; (5) \begin{pmatrix} 1 & -a & & & \\ & 1 & -a & & \\ & & 1 & \ddots & \\ & & & \ddots & -a \\ & & & & 1 \end{pmatrix};$$

$$(6) \begin{pmatrix} -\frac{n-2}{n-1} & \frac{1}{n-1} & \cdots & \frac{1}{n-1} \\ \frac{1}{n-1} & -\frac{n-2}{n-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \frac{1}{n-1} \\ \frac{1}{n-1} & \frac{1}{n-1} & \cdots & -\frac{n-2}{n-1} \end{pmatrix}.$$

$$2. (1) X = \begin{pmatrix} -3 & -1 \\ 2 & 1 \end{pmatrix}; (2) X = \begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{7}{2} & \frac{1}{2} \end{pmatrix}; (3) X = \begin{pmatrix} \frac{32}{41} & \frac{20}{41} \\ -\frac{18}{41} & -\frac{1}{41} \\ \frac{15}{41} & \frac{35}{41} \end{pmatrix}.$$

$$3. X = \begin{pmatrix} 1 & \frac{3}{2} & \frac{3}{2} \\ 0 & \frac{1}{4} & \frac{3}{4} \\ 1 & 0 & 0 \end{pmatrix}.$$

$$5. (1) \begin{pmatrix} o & B^{-1} \\ A^{-1} & 0 \end{pmatrix}; (2) \begin{pmatrix} B^{-1}CA^{-1} & B^{-1} \\ A^{-1} & 0 \end{pmatrix}.$$

$$6. f(A) = \begin{pmatrix} 1 & C_n^1 & C_n^2 & C_n^3 \\ & 1 & C_n^1 & C_n^2 \\ & & 1 & C_n^1 \\ & & & 1 \end{pmatrix}, f(A)^{-1} = \begin{pmatrix} 1 & -C_n^2 & (C_n^1)^2 - C_n^2 & -(C_n^1)^3 + 2C_n^1C_n^2 - C_n^3 \\ & 1 & -C_n^1 & (C_n^1)^2 - C_n^2 \\ & & 1 & -C_n^1 \\ & & & 1 \end{pmatrix}.$$

9. (1) 线性无关; (2) 线性相关.

14. 极大线性无关组可取 $\alpha_1, \alpha_3, \alpha_4$, 且有 $\alpha_2 = 3\alpha_1 + \alpha_3, \alpha_5 = 2\alpha_1$.

15. $\beta_1 = (1 \ 0 \ 1)^T, \beta_2 = \frac{1}{2} (1 \ 2 \ -1)^T, \beta_3 = \frac{1}{3} (1 \ -1 \ -1)^T$.

16. $\alpha_3 = \pm \frac{1}{3\sqrt{5}} (-2 \ -4 \ 5)^T$.

18. 当 $\lambda = 3$ 时, $R(A) = 2$; 当 $\lambda \neq 3$ 时, $R(A) = 3$.

19. (1) $(\alpha, \beta) = \frac{1}{2}, (\beta, \gamma) = -5, (\alpha, \gamma) = 0$;

(2) $\frac{1}{\sqrt{6}} (1 \ 2 \ 1)^T$;

(3) $\langle \alpha, \beta \rangle = \arccos \frac{1}{3\sqrt{14}}$.