

习题三

1. (1) $a = \frac{1}{3}$;

$$(2) F(x, y) = \begin{cases} 0, & x < 0 \text{ 或 } y < 0, \\ \frac{x^3}{3}y + \frac{1}{12}x^2y^2, & 0 \leq x < 1, 0 \leq y < 2, \\ \frac{y}{3} + \frac{1}{12}y^2, & x \geq 1, 0 \leq y < 2, \\ \frac{2x^3}{3} + \frac{1}{3}x^2, & 0 \leq x < 1, y \geq 2, \\ 1, & x \geq 1, y \geq 2; \end{cases}$$

(3) $P\{X + Y \leq 1\} = \frac{7}{72}, \quad P\{X + Y \leq 2.3\} = 0.7389$

2. (1) $f(x, y) = \begin{cases} \frac{1}{(b-a)(d-c)}, & a < x < b, c < y < d, \\ 0, & \text{其他,} \end{cases}$

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a < x < b, \\ 0, & \text{其他,} \end{cases} \quad f_Y(y) = \begin{cases} \frac{1}{(d-c)}, & c < y < d, \\ 0, & \text{其他;} \end{cases}$$

(2) X 与 Y 是相互独立的.

3. (1) $f_X(x) = \begin{cases} \int_0^{+\infty} \ln^2 3 \times 3^{-x-y} dy = \ln 3 \times 3^{-x}, & x > 0, \\ 0, & \text{其他;} \end{cases}$

$$f_Y(y) = \begin{cases} \int_0^{+\infty} \ln^2 3 \times 3^{-x-y} dx = \ln 3 \times 3^{-y}, & y > 0, \\ 0, & \text{其他.} \end{cases}$$

(2) X 与 Y 是相互独立的.

4. (1) $f_X(x) = \begin{cases} \frac{1}{2}, & 0 \leq x \leq 2, \\ 0, & \text{其他;} \end{cases} \quad f_Y(y) = \begin{cases} \frac{1}{2}, & 0 \leq y \leq 2, \\ 0, & \text{其他;} \end{cases}$

$$f(x, y) = \begin{cases} \frac{1}{4}, & 0 \leq x \leq 2 \quad 0 \leq y \leq 2, \\ 0, & \text{其他.} \end{cases}$$

(2) $\frac{9}{32}$.

5. (1)

$X \backslash Y$	-1	0	1
0	$\frac{1}{4}$	0	$\frac{1}{4}$
1	0	$\frac{1}{2}$	0

(2) X, Y 不独立.

$$6. (1) f_X(x) = \begin{cases} x, & 0 \leq x < 1, \\ 2 - x, & 1 \leq x < 2, \\ 0, & x < 0 \text{ 或 } x \geq 2, \end{cases} \quad f_Y(y) = \begin{cases} 2 - 2y, & 0 < y < 1, \\ 0, & \text{其他.} \end{cases}$$

$$(2) f_{XY}(x|y) = \frac{f(x,y)}{f_Y(y)} = \begin{cases} \frac{1}{2-2y}, & y < x < 2-y, 0 < y < 1, \\ 0, & \text{其他.} \end{cases}$$

$$f_{YX}(y|x) = \frac{f(x,y)}{f_X(x)} = \begin{cases} \frac{1}{x}, & 0 < x < 1, y < x, \\ \frac{1}{2-x}, & 1 \leq x < 2, 0 < y < 2-x, \\ 0, & \text{其他;} \end{cases}$$

$$f_{YX}(y|x=1.5) = \begin{cases} \frac{1}{2-1.5}, & 0 < y < 2-1.5, \\ 0, & \text{其他.} \end{cases} = \begin{cases} 2, & 0 < y < 0.5, \\ 0, & \text{其他.} \end{cases}$$

(3) (X, Y) 不相互独立.

(4) 0.6

$$(5) F_{XY}(x|y) = \begin{cases} 0, & x < y, \\ \frac{x-y}{2-2y}, & y \leq x < 2-y, \\ 1, & x \geq 2-y. \end{cases}$$

$$7. (1) f(x, y) = \frac{6}{\pi^2} \cdot \frac{1}{4+x^2} \cdot \frac{1}{9+y^2};$$

$$(2) f_x(x) = \frac{2}{\pi} \frac{1}{4+x^2}, f_y(y) = \frac{3}{\pi} \cdot \frac{1}{9+y^2}.$$

$$8. (1) \text{常数 } A = 2; (2) F(x, y) = \begin{cases} (1 - e^{-2x})(1 - e^{-y}), & x > 0, y > 0, \\ 0, & \text{其他.} \end{cases} (3) \frac{1}{3}.$$

$$9. (1) k = \frac{1}{8}; (2) \frac{3}{8}; (3) \frac{27}{32}; (4) \frac{2}{3}.$$

$$10. (1) f(x, y) = \begin{cases} \frac{1}{2} e^{-\frac{y}{2}}, & 0 < x < 1, y > 0, \\ 0, & \text{其他;} \end{cases}; (2) 0.1445.$$

$$11. (1) C = \frac{3}{\pi R^3}; (2) C = \frac{3r^2}{R^2} \left[1 - \frac{2r}{3R} \right]$$

$$13. F_Z(z) = \begin{cases} 0, & z < 0, \\ 1 - e^{-z} - ze^{-z}, & z \geq 0. \end{cases}$$

$$14. (1) f_Z(z) = \begin{cases} -\ln z, & 0 < z < 1, \\ 0, & \text{其他;} \end{cases}$$

$$(2) f_Z(z) = \begin{cases} 1, & 0 < z < 1, \\ 0, & \text{其他.} \end{cases}$$

$$15. \frac{5}{7}.$$

$$16. (1)$$

$\begin{matrix} Y \\ \backslash X \end{matrix}$	1	2	3	$p_{i\cdot}$
1	$\frac{1}{9}$	0	0	$\frac{1}{9}$
2	$\frac{2}{9}$	$\frac{1}{3}$	0	$\frac{1}{3}$
3	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{5}{9}$
$p_{\cdot j}$	$\frac{5}{9}$	$\frac{3}{9}$	$\frac{1}{9}$	1

(2) $\frac{1}{3}$.

17. (1) $\frac{3}{4}$; (2) $\frac{3}{4}$.

18. (1) $C_n^m p^m (1-p)^{n-m}, 0 \leqslant m \leqslant n, n = 0, 1, 2, \cdots$

(2) $C_n^m p^m (1-p)^{n-m} \frac{e^{-\lambda}}{n!} \lambda^n, 0 \leqslant m \leqslant n, n = 0, 1, 2, \cdots$

19.

$$f_Z(z) = \begin{cases} \frac{1}{2z^2}, & z \geqslant 1, \\ \frac{1}{2}, & 0 < z < 1, \\ 0, & \text{其他.} \end{cases}$$

20. (1)0.5; $\frac{1}{3}$.

(2)

$V = \max(X, Y)$	0	1	2	3	4	5
P	0	0.04	0.16	0.28	0.24	0.28

(3)

$U = \min(X, Y)$	0	1	2	3
P	0.28	0.30	0.25	0.17

(4)

$W = X + Y$	0	1	2	3	4	5	6	7	8
P	0	0.02	0.06	0.13	0.19	0.24	0.19	0.12	0.05