

习题四

一、填空题

1. 36. 2. $\frac{1}{2}$, 5. 3. 1.

二、计算题

1. (1) $e^{-\frac{1}{8}} - e^{-\frac{1}{2}}$; (2) $\sqrt{2\pi}$.

2. (1) $\frac{1}{3}$; (2) $\frac{2}{\sqrt{\pi}}$.

3. (1) $a = 0.2$, $b = 0.1$, $c = 0.1$.

(2) Z 的概率分布为

Z	-2	-1	0	1	2
P	0.2	0.1	0.3	0.3	0.1

(3) $P\{X = Z\} = 0.4$.

4. $E(Y^2) = 5$.

5. $E(X) = \frac{1}{p}$, $D(X) = \frac{1-p}{p^2}$.

6. $E(Z) = \frac{n^2 - 1}{3n}a$.

7. (1) Y 的分布列为 $P(Y = k) = C_{100}^k (0.6826)^k (0.3174)^{100-k}$;

(2) $EY = 100 \times 0.6826 = 68.26$, $DY = 68.26 \times 0.3174 = 21.6657$.

8. (1) $E(X) = 0$, $D(X) = 2$.

(2) $\text{Cov}(X, |X|) = 0$, X 与 $|X|$ 不相关;

(3) 随机变量 X 与 $|X|$ 不独立.

9. X 与 Y 不相关不独立.

10. (1) $X + Y$ 的概率分布.

S	0	1	2	3
$P\{X + Y = S\}$	0.10	0.40	0.35	0.15

(2) 0.25;

(3) $\rho_{XY} = -0.0676$.

11. (1) $E(Z) = \frac{1}{3}$; $D(Z) = 3$;

(2) $\rho_{XZ} = 0$;

(3) X 与 Z 相互独立.

12. (1) 常数 $a = \frac{1}{4}$, $b = -\frac{1}{4}$, $c = 1$;

(2) 方差 $D(X) = \frac{2}{3}$.

$$(3)E(Y) = \frac{1}{4} \left(e^2 - 1 \right)^2, D(Y) = \frac{1}{4} e^2 \left(e^2 - 1 \right)^2.$$

$$13. \quad (1)$$

X_1		0	1
		0.1	0.8
X_2	0	0.1	0.8
	1	0.1	0

$$(2)P = -\frac{2}{3}.$$

$$14. \quad \frac{\sigma_2^2 - \rho \sigma_1 \sigma_2}{\sigma_1^2 + \sigma_2^2 - 2 \rho \sigma_1 \sigma_2} X_1 + \frac{\sigma_1^2 - \rho \sigma_1 \sigma_2}{\sigma_1^2 + \sigma_2^2 - 2 \rho \sigma_1 \sigma_2} X_2.$$

$$15. \quad E(X) = \frac{7}{6}, E(Y) = \frac{7}{6}, \operatorname{Cov}(X, Y) = -\frac{1}{36}, \rho_{XY} = -\frac{1}{11}, D(X + Y) = \frac{5}{9}.$$

$$16. \quad (1) f_V(v) = \begin{cases} 2e^{-2v}, & v > 0 \\ 0, & v \leqslant 0 \end{cases}; \quad (2) E(U + V) = 2.$$

$$17. \quad (1)0.894\,4; (2)0.137\,9.$$

$$18. \quad P\{X = k\} = C_{100}^k 0.2^k 0.8^{100-k}, \, k = 1, 2, \cdots, 100; \quad 0.927.$$

$$19. \quad n \geqslant 14.$$

$$20. \quad 0.1814.$$

$$21. \quad (1)0.18; \quad (2)443.$$