

习题 1.5

1. 略 2. $\frac{13}{48}$

3. $\frac{m}{m+n \cdot 2^r}$

解: 设 $T = \{\text{将硬币抛 } r \text{ 次每次都出现国徽}\}$,

$A = \{\text{取到的硬币是正品}\}$.

由题设 $P(A) = \frac{m}{m+n}$, $P(\bar{A}) = \frac{n}{m+n}$,

$$P(T|A) = \frac{1}{2^r}, \quad P(T|\bar{A}) = 1,$$

由贝叶斯公式, 所求概率为

$$\begin{aligned} P(A|T) &= \frac{P(T|A)P(A)}{P(T|A)P(A) + P(T|\bar{A})P(\bar{A})} \\ &= \left(\frac{1}{2^r} \times \frac{m}{m+n} \right) \bigg/ \left(\frac{1}{2^r} \times \frac{m}{m+n} + \frac{m}{m+n} \right) \\ &= \frac{m}{m+n \cdot 2^r} \end{aligned}$$