习题 2.4

1. 略；

2. (1) $A=3$；(2) 0.740 8；(3) $F(x)=\begin{cases}1-e^{-3x}, & x\geqslant 0,\\ 0, & x<0.\end{cases}$

3. (1) $A=1$, $B=-1$；(2) $f(x)=\begin{cases}xe^{-\frac{x^2}{2}}, & x>0,\\ 0, & x\leqslant 0.\end{cases}$ (3) 0.4712.

4. 0.6.

5. 0.636 8.

6. 0.977 2；0.309 4；0.682 6.

解：这里 $\mu=1$, $\sigma=2$, 故

$$F(5)=P\{X\leqslant 5\}=P\left\{\frac{X-1}{2}\leqslant\frac{5-1}{2}\right\}=\Phi\left(\frac{5-1}{2}\right)=\Phi(2)\xlongequal{\text{查表得}}0.9772,$$

$$P\{0<X\leqslant 1.6\}=\Phi\left(\frac{1.6-1}{2}\right)-\Phi\left(\frac{0-1}{2}\right)=\Phi(0.3)-\Phi(-0.5)$$

$$=0.6179-[1-\Phi(0.5)]=0.6179-(1-0.6915)=0.3094;$$

$$P\{|X-1|\leqslant 2\}=P\{-1\leqslant X\leqslant 3\}=P\left\{-1\leqslant\frac{X-1}{2}2\text{—}\leqslant 1\right\}$$

$$=\Phi(1)-\Phi(-1)=2\Phi(1)-1=2\times 0.8413-1=0.6826.$$

7. 78 分.

解：设获奖分数线为 x_0, 则求使 $P\{X\geqslant x_0\}=0.1$ 成立的 x_0.

$$P\{X\geqslant x_0\}=1-P\{X<x_0\}=1-F(x_0)=1-\Phi\left(\frac{x_0-65}{10}\right)=0.1,$$

即 $\Phi\left(\frac{x_0-65}{10}\right)=0.9$, 查表得 $\frac{x_0-65}{10}=1.29$, 解得 $x_0=77.9$, 故分数线可定为 78 分.

8. 0.022 8, $d\geqslant 81.163\ 5$.

解：(1) 所求概率为

$$P\{X<89\}=P\left\{\frac{X-90}{0.5}<\frac{89-90}{0.5}\right\}=\Phi\left(\frac{89-90}{0.5}\right)=\Phi(-2)$$

$$=1-\Phi(2)=1-0.9772=0.0228.$$

(2) 按题意需求 d 满足

$$0.99\leqslant P\{X\geqslant 80\}=P\left\{\frac{X-d}{0.5}\geqslant\frac{80-d}{0.5}\right\}=1-P\left\{\frac{X-d}{0.5}<\frac{80-d}{0.5}\right\}=1-\Phi\left(\frac{80-d}{0.5}\right),$$

即 $\Phi\left(\frac{80-d}{0.5}\right)\leqslant 1-0.99=1-\Phi(2.325)=\Phi(-2.325)$,

亦即 $\frac{80-d}{0.5}\leqslant -2.325$, 故需 $d\geqslant 81.1635$.