

习题 3.2

1. (1)

X	1	2	3
$P_{i \cdot}$	$\frac{1}{9}$	$\frac{2}{3}$	$\frac{2}{9}$

Y	0	1	2	3
$P_{\cdot j}$	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{6}{27}$	$\frac{1}{27}$

(2)

$Y X = 1$	0	1	2	3
p	$\frac{2}{3}$	0	0	$\frac{1}{3}$

$X Y = 0$	1	2	3
p	$\frac{1}{4}$	$\frac{3}{4}$	0

$$(3) P\{X = 3 | Y = 2\} = 0, P\{Y = 2 | X = 3\} = 0.$$

2.

$$f_{X|Y}(x | y) = \frac{f(x, y)}{f_Y(y)} = \begin{cases} \frac{1}{1-y}, & y < x < 1, \\ \frac{1}{1+y}, & -y < x < 1, \\ 0, & \text{其他.} \end{cases}$$

$$f_{Y|X}(y | x) = \frac{f(x, y)}{f_X(x)} = \begin{cases} \frac{1}{2x}, & |y| < x < 1, \\ 0, & \text{其他.} \end{cases}$$

3. $\frac{47}{64}$.

解: (X, Y) 的概率密度为

$$f(x, y) = f_{X|Y}(x | y) \cdot f_Y(y) = \begin{cases} 15x^2y, & 0 < x < y < 1, \\ 0, & \text{其他.} \end{cases}$$

$$\begin{aligned} P\left(X > \frac{1}{2}\right) &= \int_{\frac{1}{2}}^1 \int_x^1 15x^2y \, dy \, dx \\ &= \int_{\frac{1}{2}}^1 15x^2 \cdot \frac{1-x^2}{2} \, dx = \frac{47}{64}. \end{aligned}$$

