习题 3. 3

1. (1) X 和 Y 的边缘分布如下表

$Y \backslash X$	2	5	8	$P\{Y=y_i\}$
0. 4	0. 15	0. 30	0. 35	0. 8
0. 8	0. 05	0. 12	0. 03	0. 2
$P\{X=x_j\}$	0. 2	0. 42	0. 38	

(2) 不独立.

2. (1) X, Y 相互独立; (2) $e^{-2.4}\approx 0.091$

解: (1) $F_X(x)=F(x,+\infty)=\begin{cases}1-e^{-0.01x} & x\geqslant 0\\ 0 & x<0\end{cases}$

$F_Y(y)=F(+\infty,y)=\begin{cases}1-e^{-0.01y} & y\geqslant 0\\ 0 & y<0\end{cases}$

易证 $F_X(x)F_Y(y)=F(x,y)$, 故 X, Y 相互独立.

(2) 由(1) X, Y 相互独立

$$\begin{aligned}P\{X>120,\ Y>120\}&=P\{X>120\}\cdot P\{Y>120\}\\&=[1-P\{X\leqslant 120\}]\cdot[1-P\{Y\leqslant 120\}]\\&=[1-F_X(120)][1-F_Y(120)]=e^{-2\cdot 4}=0.091.\end{aligned}$$

3. (1) $f(x,y)=\begin{cases}\dfrac{3}{4}, & (x,y)\in D\\ 0, & \text{其他}\end{cases}$;

$f_X(x)=\begin{cases}\dfrac{3}{2}\sqrt{x}, & 0<x<1,\\ 0, & \text{其他.}\end{cases}$ $f_Y(y)=\begin{cases}\dfrac{3}{4}(1-y^2), & -1<y<1,\\ 0, & \text{其他.}\end{cases}$

X, Y 不相互独立.

(3) $P\{X<\frac{1}{2}\}=\frac{\sqrt{2}}{4}$; $P\{Y<\frac{1}{2}\}=\frac{27}{32}$; $P\{X<\frac{1}{2},Y<\frac{1}{2}\}=\frac{5}{32}-\frac{1}{4\sqrt{2}}$.

解: (1) 区域 D: $0\leqslant x\leqslant 1$, $y^2\leqslant x$ 的面积为

$$A=2\int_0^1\sqrt{x}\,dx=\frac{4}{3},$$

依题意有: $f(x,y)=\begin{cases}\dfrac{1}{A}, 0\leqslant x\leqslant 1, y^2\leqslant x\\ 0, \text{其它}\end{cases}=\begin{cases}\dfrac{3}{4}, 0\leqslant x\leqslant 1, y^2\leqslant x\\ 0, \text{其它}\end{cases}$

(2) $f_X(x)=\int_{-\infty}^{+\infty}f(x,y)\,dy=\begin{cases}\int_{\sqrt{x}}^{\sqrt{x}}\dfrac{3}{4}dy=\dfrac{3}{2}\sqrt{x}, 0\leqslant x\leqslant 1\\ 0, \text{其它}\end{cases}$

$$f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)\,dy=\begin{cases}\int_{y^2}^{1}\frac{3}{4}dx=\frac{3}{4}(1-y^2),\ -1\leqslant y\leqslant 1\\ 0,\ 其它\end{cases},$$

又 $\because f_X(x)\cdot f_Y(y)\neq f(x,y)$

$\therefore$ X，Y 不相互独立.

(3) $P\{X<\frac{1}{2}\}=\int_0^{\frac{1}{2}}f_X(x)\,dx=\int_0^{\frac{1}{2}}\frac{3}{2}\sqrt{x}\,dx=\left(\frac{1}{2}\right)^{\frac{3}{2}}=\frac{1}{2\sqrt{2}}$;

$P\left\{Y<\frac{1}{2}\right\}=\int_{-1}^{\frac{1}{2}}f_Y(y)\,dy=\int_{-1}^{\frac{1}{2}}\frac{3}{4}(1-y^2)\,dy=\frac{3}{4}\left(y-\frac{y^3}{3}\right)\Big|_{-1}^{\frac{1}{2}}=\frac{27}{32}$;

$P\left\{X<\frac{1}{2},Y<\frac{1}{2}\right\}=\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{2}}dy\int_{y^2}^{\frac{1}{2}}\frac{3}{4}dx=\frac{5}{32}+\frac{1}{4\sqrt{2}}$.