习题 5.1

1. $\overline{X} = 3.59$, $S^2 = 2.881$.

2. (1)(3)(4)(6) 是统计量, (2)(5) 不是统计量.

3. $f(x_1, x_2, x_3, x_4) = \prod_{i=1}^{4} f(x_i) = \begin{cases} \frac{1}{\theta}^4 e^{-\frac{1}{\theta}\sum_{i=1}^{4} x_{ii}}, & x_i > 0, \\ 0, & x_i \leqslant 0. \end{cases} \quad (\frac{1}{\theta} > 0).$

解: 联合概率密度 $f(x_1, x_2, x_3, x_4) = \prod_{i=1}^{4} f(x_i) = \begin{cases} \lambda^4 e^{-\lambda\sum_{i=1}^{4} x_{ii}}, & x_i > 0, (\lambda > 0) \\ 0, & x_i \leqslant 0, \end{cases}$

4. $\frac{a+b}{2}$, $\frac{1}{12n}(b-a)^2$.

解: 由题设知 $E(X_i) = \frac{a+b}{2}$, $D(X_i) = \frac{1}{12}(b-a)^2$

$\overline{X} = \frac{X_1 + X_2 + \cdots X_n}{n} = \frac{1}{n}\sum_{i=1}^{n} X_i$, 则 $E(\overline{X}) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}\sum_{i=1}^{n} \frac{a+b}{2} = \frac{a+b}{2}$,

$D(\overline{X}) = \frac{1}{n^2}\sum_{i=1}^{n} D(X_i) = \frac{1}{n^2}\sum_{i=1}^{n} \frac{1}{12}(b-a)^2 = \frac{1}{12n}(b-a)^2$

5. $F^*_{(x)} = \begin{cases} 0, & x \leqslant 1, \\ \frac{1}{8}, & 1 < x \leqslant 2, \\ \frac{3}{8}, & 2 < x \leqslant 3, \\ \frac{6}{8}, & 3 < x \leqslant 4, \\ \frac{7}{8}, & 4 < x \leqslant 5, \\ 1, & x > 5. \end{cases}$