

习题 5.2

1. (1) 31.41, 10.851; (2) 1.725, -1.725, ± 2.086 .

2. (1) $t(2)$; (2) $t(n-1)$; (3) $F(3, n-3)$.

3. $n, 2n$.

4. 0.1.

解: $\because X_i \sim N(0, 0.3^2)$,

$$\therefore \frac{X_i}{0.3} \sim N(0, 1),$$

$$\text{从而 } \sum_{i=1}^{10} \frac{X_i^2}{0.09} \sim \chi^2(10),$$

$$P\left\{\sum_{i=1}^{10} X_i^2 > 1.44\right\} = P\left\{\sum_{i=1}^{10} \frac{X_i^2}{0.09} > \frac{1.44}{0.09} = 16\right\} = 0.1$$

5. $a = \frac{1}{10}$, $b = \frac{1}{50}$, 自由度 2.

解: 要使 $Y = a(2X_1 - X_2)^2 + b(3X_3 + 4X_4)^2$ 服从 χ^2 分布,

$\sqrt{a}(2X_1 - X_2)$, $\sqrt{b}(3X_3 + 4X_4)$ 必为标准正态分布, 且自由度为 2.

$$\text{由 } D[\sqrt{a}(2X_1 - X_2)] = 1, \text{ 得 } 5a \cdot 2 = 1 \Rightarrow a = \frac{1}{10},$$

$$\text{由 } D[\sqrt{b}(3X_3 + 4X_4)] = 1, \text{ 得 } 25b \cdot 2 = 1 \Rightarrow b = \frac{1}{50}.$$

6. $T \sim t(4)$, $t_0 = 4.6041$.

解: $X \sim N(2, 1) \Rightarrow X - 2 \sim N(0, 1)$,

$$Y_i \sim N(0, 4) \Rightarrow \frac{Y_i}{2} \sim N(0, 1), \text{ 则 } \sum_{i=1}^4 \left(\frac{Y_i}{2}\right)^2 \sim \chi^2(4),$$

$$\text{故 } T = \frac{4(X-2)}{\sqrt{\sum_{i=1}^4 Y_i^2}} = \frac{X-2}{\sqrt{\sum_{i=1}^4 \left(\frac{Y_i}{2}\right)^2}} \sim t(4)$$

由 $P\{|T| > t_0\} = 0.01$, 反查 t 分布表, 得 $t_0 = 4.6041$.