Appendix A. Detailed derivation of frost heave pressure

The p in cracks is affected by freezing time, temperature and water volume, and its calculation formula is:

$$p_i\left(t\right)=\frac{L(-T_\mathrm{c})}{T_\mathrm{a}\theta_i}\left(1-\mathrm{e}^{-\frac{t}{\tau}}\right)+p_0\mathrm{e}^{-\frac{t}{\tau}} \tag{A1}$$

$$\tau=\left(\frac{8}{3\pi}\right)\left(\frac{1-v}{E'}\right)\left(\frac{gw\theta_\mathrm{L}}{\theta_i^2}\right)R_\mathrm{f} \tag{A2}$$

where $p_i\left(t\right)$ is the frost heaving pressure at time t; $L(-T_\mathrm{c})$ is the latent heat of transformation when the temperature of crack wall is T_c; T_a is the absolute temperature, 273.15 K; P_0 is the frost heave pressure at the initial stage of freezing; θ_i is relative volume content of ice; v is the Poisson ratio of rock; E' is the shear modulus of rock; g is the acceleration of gravity; θ_L is the volume content of water; R_f is the total choke flow.

According to the literature, it could also be obtained:

$$\frac{L\left(-T_\mathrm{c}\right)}{T_\mathrm{a}\theta_i}=1.1\times\left(-T_\mathrm{c}\right) \tag{A3}$$

Assuming that the total volume of rock is V, the volume contents of the internal matrix, water and ice are shown in Eq. (A4). If the initial porosity of the rock is δ, the volume content of unfrozen water is χ, then:

$$\begin{cases}\theta_\mathrm{s}=V_\mathrm{s}/V\\ \theta_\mathrm{L}=V_\mathrm{L}/V\\ \theta_i=V_i/V\end{cases} \tag{A4}$$

$$\begin{cases}\theta_\mathrm{s}=1-\delta\\ \theta_\mathrm{L}=\delta\chi\\ \theta_i=\delta\left(1-\chi\right)\end{cases} \tag{A5}$$

where V_s is the volume of rock matrix; θ_s is the volume content of rock matrix; V_L is the volume of water; θ_L is the volume content of water.

According to the existing research results, the relationship between χ and temperature can be expressed by a smooth continuous function. When the temperature is within −1 °C–0 °C, χ decreases sharply with the decrease of temperature, and when the temperature is lower than −1 °C, χ gradually tends to be stable. Therefore, the following step function is constructed as:

$$H\left[T_\mathrm{c}-\Delta T\right]=\begin{cases}0, T_\mathrm{c}-T_\mathrm{a}\leq-\Delta T\\ 1, T_\mathrm{c}-T_\mathrm{a}>\Delta T\end{cases} \tag{A6}$$

where T_c is the crack wall temperature, 0.5 °C; T_a is the phase transition temperature, 0 °C; ΔT is the temperature step, 0.5 °C.

When the temperature is lower than −0.5 °C, take χ as 0.07, when the temperature is higher than 0.5 °C, take χ as 1. Therefore, χ can be expressed as:

$$\chi=0.93H\left[T_\mathrm{c}-\Delta T\right]+0.07 \tag{A7}$$

The work of frost heaving pressure in freezing time is:

$$H = 4c\frac{\Delta w}{t_i}\int_0^{t_i} p_i(t)\mathrm{d}t \tag{A8}$$

where t_i is the freezing time.

From Eqs. (5) and (14), the equivalent frost heave pressure can be obtained as:

$$p = \frac{\int_0^{t_i} p_i(t)\mathrm{d}t}{t_i} \tag{A9}$$

Referring to the strength factor solution of mode I crack, the stress intensity factor of infinite plane crack under uniform pressure is:

$$K_{\mathrm{I}} = p\sqrt{2\pi c} \tag{A10}$$

The calculation formula of fracture energy release rate G is:

$$G = \frac{K_{\mathrm{I}}^2(1-v^2)}{E} \tag{A11}$$