Supplementary materials

$$f_1 = \frac{1}{2}\left[\mathrm{e}^{(\theta-\theta_0)\tan\varphi'} + \frac{r_0'}{r_0}\mathrm{e}^{-(\theta-\theta_0)\tan\varphi'} \right] \tag{A1}$$

$$f_2 = \frac{1}{2}\left[\mathrm{e}^{(\theta-\theta_0)\tan\varphi'} - \frac{r_0'}{r_0}\mathrm{e}^{-(\theta-\theta_0)\tan\varphi'} \right] \tag{A2}$$

$$f_3 = \frac{\sin\theta_0}{\sin\theta} - f_1 \tag{A3}$$

$$f_4 = \frac{\sin\theta_0 \sin\left(\theta_\mathrm{B} + \beta_1\right)}{\sin\theta_\mathrm{B} \sin\left(\theta + \beta_1\right)} - f_1 \tag{A4}$$

$$f_5 = \frac{\sin\theta_0 + \delta_1\varepsilon_1}{\sin\theta} - f_1 \tag{A5}$$

$$f_6 = \mathrm{e}^{(\theta-\theta_0)\tan\varphi'} \frac{\sin\theta_0 \sin\left(\theta_\mathrm{h} + \beta_2\right)}{\sin\theta_\mathrm{B} \sin\left(\theta + \beta_2\right)} - f_1 \tag{A6}$$

$$f_7 = \mathrm{e}^{(\theta-\theta_0)\tan\varphi'} \sin\theta - \sin\theta_0 \tag{A7}$$

$$f_8 = \frac{\left(3\tan\varphi'\cos\theta_\mathrm{h} + \sin\theta_\mathrm{h}\right)\mathrm{e}^{3(\theta-\theta_0)\tan\varphi'} - 3\tan\varphi'\cos\theta_0 - \sin\theta_0}{3\left(1 + 9\tan^2\varphi'\right)} \tag{A8}$$

$$f_9 = \frac{1}{3}\varepsilon_2 \sin\theta_0 \left(\cos\theta_0 - \frac{1}{2}\varepsilon_2 \right) \tag{A9}$$

$$f_{10} = \frac{1}{3}\delta_1\varepsilon_1 \left(\cos\theta_0 - \varepsilon_2 + \sin\theta_0 \cot\beta_1\right)\left(\cos\theta_0 - \varepsilon_2 - \frac{1}{2}\delta_1\varepsilon_1 \cot\beta_1 \right) \tag{A10}$$

$$f_{11} = \frac{1}{3}\lambda_\mathrm{a}\varepsilon_1 \left(\sin\theta_0 + \delta_1\varepsilon_1\right)\left(\cos\theta_0 - \varepsilon_2 - \delta_1\varepsilon_1\cot\beta_1 - \frac{1}{2}\lambda_\mathrm{a}\varepsilon_1 \right) \tag{A11}$$

$$f_{12} = \frac{1}{3}\delta_2\varepsilon_1 \mathrm{e}^{(\theta-\theta_0)\tan\varphi'}\left[\mathrm{e}^{(\theta-\theta_0)\tan\varphi'}\cos\theta_\mathrm{h} + \frac{1}{2}\delta_2\varepsilon_1\cot\beta_2 \right] \tag{A12}$$

$$f_{13} = \mathrm{e}^{(\alpha-\theta_0)\tan\varphi'} \frac{\sin\alpha}{\sin\theta} - f_1 \tag{A13}$$

$$\begin{aligned} g_1 = {} & \int_{\theta_0}^{\theta_\mathrm{B}} f_2\left(2f_1^2 + f_2^2\right)\arccos\left(f_3 / f_2\right) + f_2\left(4f_1 + f_3\right)\sqrt{f_2^2 - f_3^2}\,\mathrm{d}\theta + \\ & \int_{\theta_\mathrm{B}}^{\theta_\mathrm{C}} f_2\left(2f_1^2 + f_2^2\right)\arccos\left(f_4 / f_2\right) + f_2\left(4f_1 + f_4\right)\sqrt{f_2^2 - f_4^2}\,\mathrm{d}\theta + \\ & \int_{\theta_\mathrm{C}}^{\theta_\mathrm{D}} f_2\left(2f_1^2 + f_2^2\right)\arccos\left(f_5 / f_2\right) + f_2\left(4f_1 + f_5\right)\sqrt{f_2^2 - f_5^2}\,\mathrm{d}\theta + \\ & \int_{\theta_\mathrm{D}}^{\theta_\mathrm{h}} f_2\left(2f_1^2 + f_2^2\right)\arccos\left(f_6 / f_2\right) + f_2\left(4f_1 + f_6\right)\sqrt{f_2^2 - f_6^2}\,\mathrm{d}\theta \end{aligned} \tag{A14}$$

$$g_2=\int_{\theta_0}^{\theta_B} f_2 f_7\left(2f_1^2+f_2^2\right)\arccos\left(f_3/f_2\right)+f_2 f_7\left(4f_1+f_3\right)\sqrt{f_2^2-f_3^2}\,\mathrm{d}\theta+$$
$$\int_{\theta_B}^{\theta_C} f_2 f_7\left(2f_1^2+f_2^2\right)\arccos\left(f_4/f_2\right)+f_2 f_7\left(4f_1+f_3\right)\sqrt{f_2^2-f_4^2}\,\mathrm{d}\theta+$$
$$\int_{\theta_C}^{\theta_D} f_2 f_7\left(2f_1^2+f_2^2\right)\arccos\left(f_5/f_2\right)+f_2 f_7\left(4f_1+f_3\right)\sqrt{f_2^2-f_5^2}\,\mathrm{d}\theta+$$
$$\int_{\theta_D}^{\theta_h} f_2 f_7\left(2f_1^2+f_2^2\right)\arccos\left(f_6/f_2\right)+f_2 f_7\left(4f_1+f_3\right)\sqrt{f_2^2-f_6^2}\,\mathrm{d}\theta \tag{A15}$$

$$g_3=\int_{\theta_0}^{\theta_B}\left[\left(2f_1f_2^3-2f_1^2f_2f_3-f_2^3f_3\right)\arccos\left(\frac{f_3}{f_2}\right)+\left(2f_1^2f_2-2f_1f_2f_3+\frac{4}{3}f_2^3-\frac{1}{3}f_2f_3^2\right)\sqrt{f_2^2-f_3^2}\right]\sin\theta\mathrm{d}\theta+$$
$$\int_{\theta_B}^{\theta_C}\left[\left(2f_1f_2^3-2f_1^2f_2f_3-f_2^3f_4\right)\arccos\left(\frac{f_4}{f_2}\right)+\left(2f_1^2f_2-2f_1f_2f_4+\frac{4}{3}f_2^3-\frac{1}{3}f_2f_4^2\right)\sqrt{f_2^2-f_4^2}\right]\sin\theta\mathrm{d}\theta+$$
$$\int_{\theta_C}^{\theta_D}\left[\left(2f_1f_2^3-2f_1^2f_2f_5-f_2^3f_5\right)\arccos\left(\frac{f_5}{f_2}\right)+\left(2f_1^2f_2-2f_1f_2f_5+\frac{4}{3}f_2^3-\frac{1}{3}f_2f_5^2\right)\sqrt{f_2^2-f_5^2}\right]\sin\theta\mathrm{d}\theta+$$
$$\int_{\theta_D}^{\theta_h}\left[\left(2f_1f_2^3-2f_1^2f_2f_6-f_2^3f_6\right)\arccos\left(\frac{f_6}{f_2}\right)+\left(2f_1^2f_2-2f_1f_2f_6+\frac{4}{3}f_2^3-\frac{1}{3}f_2f_6^2\right)\sqrt{f_2^2-f_6^2}\right]\sin\theta\mathrm{d}\theta \tag{A16}$$

$$g_4=\frac{b}{H}\frac{1}{2\tan\varphi'}\left[\mathrm{e}^{(\theta_h-\theta_0)\tan\varphi'}\sin\theta_h-\sin\theta_0\right]\left[\mathrm{e}^{2(\theta_h-\theta_0)\tan\varphi'}-1\right] \tag{A17}$$

$$g_5=\frac{b}{H}\delta_1\left\{\frac{1}{3\tan\varphi'}\left[\sin\theta_h\mathrm{e}^{3(\theta_h-\theta_0)\tan\varphi'}-\sin\theta_0\right]+\right.$$
$$\left.\frac{\cos\theta_0-\cos\theta_h\mathrm{e}^{3(\theta_h-\theta_0)\tan\varphi'}}{9\tan^2\varphi'}-\frac{1}{2\tan\varphi'}\sin\theta_0\left[\mathrm{e}^{2(\theta_h-\theta_0)\tan\varphi'}-1\right]\right\} \tag{A18}$$

$$g_6=2\int_{\theta_0}^{\theta_B}\left[\left(\frac{1}{8}f_2^2f_3-\frac{1}{4}f_3^3-\frac{2}{3}f_1f_3^2-\frac{1}{2}f_1^2f_3+\frac{2}{3}f_1f_2^2\right)\times\right.$$
$$\left.\sqrt{f_2^2-f_3^2}+\left(\frac{1}{8}f_2^4+\frac{1}{2}f_1^2f_2^2\right)\arccos\left(\frac{f_3}{f_2}\right)\right]\cos\theta\mathrm{d}\theta+$$
$$2\int_{\theta_B}^{\theta_C}\left[\left(\frac{1}{8}f_2^2f_4-\frac{1}{4}f_4^3-\frac{2}{3}f_1f_4^2-\frac{1}{2}f_1^2f_4+\frac{2}{3}f_1f_2^2\right)\times\right.$$
$$\left.\sqrt{f_2^2-f_4^2}+\left(\frac{1}{8}f_2^4+\frac{1}{2}f_1^2f_2^2\right)\arccos\left(\frac{f_4}{f_2}\right)\right]\cos\theta\mathrm{d}\theta+$$
$$2\int_{\theta_C}^{\theta_D}\left[\left(\frac{1}{8}f_2^2f_5-\frac{1}{4}f_5^3-\frac{2}{3}f_1f_5^2-\frac{1}{2}f_1^2f_5+\frac{2}{3}f_1f_2^2\right)\times\right.$$
$$\left.\sqrt{f_2^2-f_5^2}+\left(\frac{1}{8}f_2^4+\frac{1}{2}f_1^2f_2^2\right)\arccos\left(\frac{f_5}{f_2}\right)\right]\cos\theta\mathrm{d}\theta+$$
$$2\int_{\theta_D}^{\theta_h}\left[\left(\frac{1}{8}f_2^2f_6-\frac{1}{4}f_6^3-\frac{2}{3}f_1f_6^2-\frac{1}{2}f_1^2f_6+\frac{2}{3}f_1f_2^2\right)\times\right.$$
$$\left.\sqrt{f_2^2-f_6^2}+\left(\frac{1}{8}f_2^4+\frac{1}{2}f_1^2f_2^2\right)\arccos\left(\frac{f_6}{f_2}\right)\right]\cos\theta\mathrm{d}\theta \tag{A19}$$

$$g_7 = \frac{b}{H}\delta_1\left(f_8 - f_9 - f_{10} - f_{11} - f_{12}\right) \tag{A20}$$

$$g_8 = \int_{\theta_0}^{\theta_h}\int_{\alpha}^{\alpha'} e^{3(\alpha-\theta_0)\tan\varphi'}\frac{\sin^2\alpha}{\sin^2\theta}\frac{\cos\left(\alpha-\varphi'\right)}{\cos\varphi'}\frac{a_h}{k_h g}\sqrt{f_2^2 - f_{13}^2}\,\mathrm{d}\alpha\mathrm{d}\theta \tag{A21}$$

$$g_9 = \frac{b}{H}\delta_1\int_{\theta_0}^{\theta_h}\int_{\alpha}^{\alpha'} e^{3(\alpha-\theta_0)\tan\varphi'}\frac{\sin^2\alpha}{\sin^2\theta}\frac{\cos\left(\alpha-\varphi'\right)}{\cos\varphi'}\frac{a_h}{k_h g}\,\mathrm{d}\alpha\mathrm{d}\theta \tag{A22}$$