Supplementary materials

Appendix 1. Calculation of the parameters in the GHB strength criterion

The calculation formulas for parameters m_b, s and ρ are as follows:

$$m_b = m_i \exp\left(\frac{\mathrm{GSI}-100}{28-14D}\right) \tag{A1}$$

$$s = \exp\left(\frac{\mathrm{GSI}-100}{9-3D}\right) \tag{A2}$$

$$\rho = 0.5 + \frac{1}{6}[\exp(-\mathrm{GSI}/15) - \exp(-20/3)] \tag{A3}$$

where m_i represents the material constant of an intact rock, ranging from 0.001 to 25; GSI denotes the geological strength index, varying between 10 to 100, where a larger value indicates less rock fragmentation; and D represents the rock weakening factor, which quantifies the degree of disturbance within the rock mass, and its value ranges from 0 to 1, where 0 corresponds to an unperturbed rock mass and 1 signifies an extremely disturbed rock mass.

Appendix 2. Description of the NS and shear stress on the slip surface at points *A* and *B*

Based on the stress functions mentioned in Eqs. (5) and (11), the equations for calculating the NS (σ_A) and shear stress (τ_A) on the SS at the lower sliding point A are as follows:

$$\sigma_A = \sigma_0\big|_{x=x_A} + \Delta\sigma_1 \tag{A4}$$

$$\tau_A = \frac{c_A + (\sigma_0\big|_{x=x_A} + \Delta\sigma_1)\tan\varphi_A}{F_s} \tag{A5}$$

where $\sigma_0\big|_{x=x_A}$ represents the fundamental NS on the SS at the lower sliding point A, which can be calculated using Eq. (2); and φ_A and c_A denote the instantaneous internal friction angle and instantaneous cohesion of the rock mass of the SS, respectively, under the action of σ_A, and these values can be calculated using Eqs. (8) and (10) correspondingly.

Based on the stress functions mentioned in Eqs. (5) and (11), the equations for calculating the NS (σ_B) and shear stress (τ_B) on the SS at the upper sliding point B are as follows:

$$\sigma_B = \sigma_0\big|_{x=x_B} + \Delta\sigma_1 + \Delta\sigma_2 \tag{A6}$$

$$\tau_B = \frac{c_B + (\sigma_0\big|_{x=x_B} + \Delta\sigma_1 + \Delta\sigma_2)\tan\varphi_B}{F_s} \tag{A7}$$

where $\sigma_0\big|_{x=x_B}$ represents the fundamental NS on the SS at the upper sliding point B, which can be calculated using Eq. (2), and $\sigma_0\big|_{x=x_B} = 0$ when there is no external load (q_x=q_y=0); and φ_B and c_B are the instantaneous internal friction angle and instantaneous cohesion of the rock mass, respectively, under the action of σ_B, and these values can be calculated using Eqs. (8) and (10) correspondingly.

Appendix 3. Calculation of unknown variables in the stress function on the slip surface

By substituting the NS function (Eq. (5)) and the shear stress function (Eq. (11)) into the overall force equilibrium equation (Eq. (15)) along the x-axes in the slope sliding body, the formula for calculating the slope safety factor (F_s) is derived as follows:

$$\frac{1}{F_\mathrm{s}}=\frac{\int_{x_A}^{x_B}(k_\mathrm{H}w-q_x)\mathrm{d}x+\int_{x_A}^{x_B}\left[\sigma_0+\Delta\sigma_1+\Delta\sigma_2\left(\frac{x-x_A}{x_B-x_A}\right)+\Delta\sigma_3\sin\left(\pi\frac{x-x_A}{x_B-x_A}\right)+\Delta\sigma_4 f_\mathrm{i}\sin\left(2\pi\frac{x-x_A}{x_B-x_A}\right)\right]s_x\mathrm{d}x}{\int_{x_A}^{x_B}c_\mathrm{i}\mathrm{d}x+\int_{x_A}^{x_B}\left[\sigma_0+\Delta\sigma_1+\Delta\sigma_2\left(\frac{x-x_A}{x_B-x_A}\right)+\Delta\sigma_3\sin\left(\pi\frac{x-x_A}{x_B-x_A}\right)+\Delta\sigma_4 f_\mathrm{i}\sin\left(2\pi\frac{x-x_A}{x_B-x_A}\right)\right]\tan\varphi_\mathrm{i}\mathrm{d}x}$$

(A8)

By implementing the stress constraint condition at point A of the SS and substituting Eqs. (A4) and (A5) into Eq. (12), the formula for calculating the variable $\Delta\sigma_1$ in the NS function on the SS is derived as follows:

$$\Delta\sigma_1=\frac{[c_A+(\sigma_0|_{x=x_A}+\Delta\sigma_1)\tan\varphi_A]\tan(\omega-\alpha_A)}{F_\mathrm{s}}-\sigma_0|_{x=x_A} \tag{A9}$$

By applying the stress constraint condition at point B of the SS and substituting Eqs. (A6) and Eq. (A7) into Eq. (13), the formula for calculating the variable $\Delta\sigma_2$ in the NS function on the SS is derived as follows:

$$\Delta\sigma_2=-\frac{c_B/\tan\phi_B}{1+\frac{F_\mathrm{s}}{\tan\alpha_B\tan\phi_B}}-\Delta\sigma_1-\sigma_0|_{x=x_B} \tag{A10}$$

By substituting the NS function (Eq. (5)) and shear stress function (Eq. (11)) into the overall force equilibrium equation (Eq. (16)) along the y-axes in the slope sliding body, the formula for calculating the variable $\Delta\sigma_3$ in the NS function on the SS is derived as follows:

$$\begin{aligned}\Delta\sigma_3=\Bigg\{&\int_{x_A}^{x_B}[(1-k_\mathrm{V})w+q_y]\mathrm{d}x-\\&\int_{x_A}^{x_B}\frac{c_\mathrm{i}+\left[\sigma_0+\Delta\sigma_1+\Delta\sigma_2\left(\frac{x-x_A}{x_B-x_A}\right)+\Delta\sigma_4 f_\mathrm{i}\sin\left(2\pi\frac{x-x_A}{x_B-x_A}\right)\right]\tan\varphi_\mathrm{i}}{F_\mathrm{s}}p_x\mathrm{d}x-\\&\int_{x_A}^{x_B}\left[\sigma_0+\Delta\sigma_1+\Delta\sigma_2\left(\frac{x-x_A}{x_B-x_A}\right)+\Delta\sigma_4 f_\mathrm{i}\sin\left(2\pi\frac{x-x_A}{x_B-x_A}\right)\right]\mathrm{d}x\Bigg\}\Bigg/\\&\left[\int_{x_A}^{x_B}\left(1+\frac{\tan\varphi_\mathrm{i}}{F_\mathrm{s}}p_x\right)\sin\left(\pi\frac{x-x_A}{x_B-x_A}\right)\mathrm{d}x\right]\end{aligned} \tag{A11}$$

Equation (A11) demonstrates that the calculation of $\Delta\sigma_3$ is linked to $\Delta\sigma_4$. If the magnitude of $\Delta\sigma_4$ is greater, the magnitude of $\Delta\sigma_3$ also increases, making it difficult for the iterative loop calculation to converge. To eliminate the computational interdependence between $\Delta\sigma_3$ and $\Delta\sigma_4$, one can set f_i as:

$$f_\mathrm{i}=\frac{1}{1+\frac{\tan\varphi_\mathrm{i}}{F_\mathrm{s}}p_x} \tag{A12}$$

After substituting Eq. (A12) into Eq. (A11), the revised formula for calculating the variable $\Delta\sigma_3$ in the NS function on the SS is obtained as:

$$\Delta\sigma_3 = \left\{ \int_{x_A}^{x_B} [(1-k_\mathrm{V})w + q_y]\mathrm{d}x - \int_{x_A}^{x_B} \frac{c_\mathrm{i} + \left[\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2\left(\dfrac{x-x_A}{x_B-x_A}\right)\right]\tan\varphi_\mathrm{i}}{F_\mathrm{s}} p_x \mathrm{d}x - \int_{x_A}^{x_B} \left[\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2\left(\frac{x-x_A}{x_B-x_A}\right)\right]\mathrm{d}x \right\} \Bigg/ \left[\int_{x_A}^{x_B}\left(1+\frac{\tan\varphi_\mathrm{i}}{F_\mathrm{s}}p_x\right)\sin\left(\pi\frac{x-x_A}{x_B-x_A}\right)\mathrm{d}x\right] \quad \text{(A13)}$$

The introduction of f_i in this context ensures the convergence of the subsequent iterative loop calculation. This convergence is made possible by the special property of the sine function, where yields a cumulative result of zero within a specific period. By utilizing this property, the computational correlation between $\Delta\sigma_3$ and $\Delta\sigma_4$ can be effectively eliminated. Furthermore, the inclusion of f_i enables the derivation of reasonable stress results on the SS. This is achieved by satisfying both the mechanical equilibrium equations of the sliding body and the stress constraint conditions at both ends of the SS. These claims are further substantiated by subsequent example analysis.

By substituting the NS function (Eq. (5)) and shear stress function (Eq. (11)) into the overall moment equilibrium equation (Eq. (17)) around the origin in the slope sliding body, the formula for calculating the variable $\Delta\sigma_4$ in the NS function on the SS is derived as:

$$\Delta\sigma_4 = \left\{ \int_{x_A}^{x_B} \left\{ [(1-k_\mathrm{V})w + q_y]x - \frac{1}{2}k_\mathrm{H} w(p+g) + q_x g \right\}\mathrm{d}x - \int_{x_A}^{x_B} \frac{c_\mathrm{i} + \left[\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2\left(\dfrac{x-x_A}{x_B-x_A}\right) + \Delta\sigma_3 \sin\left(\pi\dfrac{x-x_A}{x_B-x_A}\right)\right]\tan\varphi_\mathrm{i}}{F_s}(xp_x - p)\mathrm{d}x - \int_{x_A}^{x_B}\left[\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2\left(\frac{x-x_A}{x_B-x_A}\right) + \Delta\sigma_3\sin\left(\pi\frac{x-x_A}{x_B-x_A}\right)\right](pp_x + x)\mathrm{d}x \right\} \Bigg/ \left\{ \int_{x_A}^{x_B} f_\mathrm{i}\sin\left(2\pi\frac{x-x_A}{x_B-x_A}\right)\left[(pp_x + x) + \frac{\tan\varphi_\mathrm{i}}{F_\mathrm{s}}(xp_x - p)\right]\mathrm{d}x \right\} \quad \text{(A14)}$$