Appendix

Related parameter when 0<ρ<1

$$F_1(x)=\cos(\varphi_{12}x)\sinh(\varphi_{11}x)$$

$$F_2(x)=\cos(\varphi_{12}x)\cosh(\varphi_{11}x)$$

$$F_3(x)=\sin(\varphi_{12}x)\cosh(\varphi_{11}x)$$

$$F_4(x)=\sin(\varphi_{12}x)\sinh(\varphi_{11}x)$$

$$c_1=c_{16}=F_2(x)-\frac{\rho}{\sqrt{1-\rho^2}}F_4(x)$$

$$c_2=c_{12}=\frac{1}{2}\left(\frac{F_1(x)}{\varphi_{11}}+\frac{F_3(x)}{\varphi_{12}}\right)$$

$$c_3=c_8=-\frac{1}{2EI\varphi_{11}\varphi_{12}}F_4(x)$$

$$c_4=\frac{1}{4EI\lambda^2}\left(\frac{F_1(x)}{\varphi_{11}}-\frac{F_3(x)}{\varphi_{12}}\right)$$

$$c_5=c_{15}=\lambda^2\left(\frac{F_1(x)}{\varphi_{11}}-\frac{F_3(x)}{\varphi_{12}}\right)$$

$$c_6=c_{11}=F_2(x)+\frac{\rho}{\sqrt{1-\rho^2}}F_4(x)$$

$$c_7=-\frac{1}{2EI}\left(\frac{F_1(x)}{\varphi_{11}}+\frac{F_3(x)}{\varphi_{12}}\right)$$

$$c_9=c_{14}=2EI\lambda^4\frac{F_4(x)}{\varphi_{11}\varphi_{12}}$$

$$c_{10}=-\frac{EI\lambda^2}{2}\left(\frac{4\rho+2}{\varphi_{11}}F_1(x)+\frac{4\rho-2}{\varphi_{12}}F_3(x)\right)$$

$$c_{13}=2EI\lambda^4\left(\frac{1+2\rho}{\varphi_{11}}F_1(x)+\frac{1-2\rho}{\varphi_{12}}F_3(x)\right)$$

$$c_w=-\frac{b}{4\lambda^2EI}\int_0^x q(z)\left(\frac{F_1(x-z)}{\varphi_{11}}-\frac{F_3(x-z)}{\varphi_{12}}\right)\mathrm{d}z$$

$$c_\theta = \frac{b}{2EI}\int_0^x q(z)\frac{F_4(x-z)}{\varphi_{11}\varphi_{12}}\mathrm{d}z$$

$$c_M = -\frac{b}{2}\int_0^x q(z)\left(\frac{F_1(x-z)}{\varphi_{11}}+\frac{F_3(x-z)}{\varphi_{12}}\right)\mathrm{d}z$$

$$c_Q = -b\int_0^x q(z)\left(F_2(x-z)-\frac{\rho}{\sqrt{1-\rho^2}}F_4(x-z)\right)\mathrm{d}z$$

Related parameter when ρ=1

$$d_1 = d_{16} = \cosh(\varphi_{21}x)-\frac{\varphi_{21}x\sinh(\varphi_{21}x)}{2}$$

$$d_2 = d_{12} = \frac{1}{2}\left(x\cosh(\varphi_{21}x)+\frac{\sinh(\varphi_{21}x)}{\varphi_{21}}\right)$$

$$d_3 = d_8 = -\frac{1}{2EI}\frac{x\sinh(\varphi_{21}x)}{\varphi_{21}}$$

$$d_4 = \frac{1}{2EI}\left(\frac{-x\cosh(\varphi_{21}x)}{\varphi_{21}^2}+\frac{\sinh(\varphi_{21}x)}{\varphi_{21}^3}\right)$$

$$d_5 = d_{15} = \frac{1}{2}\left(\varphi_{21}\sinh(\varphi_{21}x)-\varphi_{21}^2x\cosh(\varphi_{21}x)\right)$$

$$d_6 = d_{11} = \frac{\varphi_{21}x\sinh(\varphi_{21}x)}{2}+\cosh(\varphi_{21}x)$$

$$d_7 = -\frac{1}{2EI}\left(x\cosh(\varphi_{21}x)+\frac{\sinh(\varphi_{21}x)}{\varphi_{21}}\right)$$

$$d_9 = d_{14} = \frac{EI}{2}\varphi_{21}^3x\sinh(\varphi_{21}x)$$

$$d_{10} = -\frac{EI}{2}\left(\varphi_{21}^2x\cosh(\varphi_{21}x)+3\varphi_{21}\sinh(\varphi_{21}x)\right)$$

$$d_{13} = \frac{EI}{2}\left(-\varphi_{21}^4x\cosh(\varphi_{21}x)+3\varphi_{21}^3\sinh(\varphi_{21}x)\right)$$

$$d_w = -\frac{b}{2EI}\int_0^x q(z)\left(\frac{-(x-z)\cosh\left[\varphi_{21}(x-z)\right]}{\varphi_{21}^2}+\frac{\sinh\left[\varphi_{21}(x-z)\right]}{\varphi_{21}^3}\right)\mathrm{d}z$$

$$d_\theta = \frac{b}{2EI}\int_0^x q(z)\frac{(x-z)\sinh\left[\varphi_{21}(x-z)\right]}{\varphi_{21}}\mathrm{d}z$$

$$d_M = -\frac{b}{2}\int_0^x q(z)\left\{(x-z)\cosh\left[\varphi_{21}(x-z)\right]+\frac{\sinh\left[\varphi_{21}(x-z)\right]}{\varphi_{21}}\right\}\mathrm{d}z$$

$$d_Q = -b\int_0^x q(z)\left\{\cosh\left[\varphi_{21}(x-z)\right]-\frac{(x-z)\varphi_{21}\sinh\left[\varphi_{21}(x-z)\right]}{2}\right\}\mathrm{d}z$$

Related parameter when ρ>1

$$H_1(x)=\cosh(\varphi_{32}x)\sinh(\varphi_{31}x)$$

$$H_2(x)=\cosh(\varphi_{32}x)\cosh(\varphi_{31}x)$$

$$H_3(x)=\sinh(\varphi_{32}x)\cosh(\varphi_{31}x)$$

$$H_4(x)=\sinh(\varphi_{32}x)\sinh(\varphi_{31}x)$$

$$e_1=e_{16}=H_2(x)-\frac{\rho}{\sqrt{\rho^2-1}}H_4(x)$$

$$e_2=e_{12}=\frac{1}{2}\left(\frac{H_1(x)}{\varphi_{31}}+\frac{H_3(x)}{\varphi_{32}}\right)$$

$$e_3=c_8=-\frac{1}{2EI\varphi_{31}\varphi_{32}}H_4(x)$$

$$e_4=\frac{1}{4EI\lambda^2}\left(\frac{H_1(x)}{\varphi_{31}}-\frac{H_3(x)}{\varphi_{32}}\right)$$

$$e_5=e_{15}=\lambda^2\left(\frac{H_1(x)}{\varphi_{31}}-\frac{H_3(x)}{\varphi_{32}}\right)$$

$$e_6=e_{11}=H_2(x)+\frac{\rho}{\sqrt{\rho^2-1}}H_4(x)$$

$$e_7=-\frac{1}{2EI}\left(\frac{H_1(x)}{\varphi_{31}}+\frac{H_3(x)}{\varphi_{32}}\right)$$

$$e_9=e_{14}=2EI\lambda^4\frac{H_4(x)}{\varphi_{31}\varphi_{32}}$$

$$e_{10}=-\frac{EI\lambda^2}{2}\left(\frac{4\rho+2}{\varphi_{31}}H_1(x)+\frac{4\rho-2}{\varphi_{32}}H_3(x)\right)$$

$$e_{13}=2EI\lambda^4\left(\frac{1+2\rho}{\varphi_{11}}H_1(x)+\frac{1-2\rho}{\varphi_{12}}H_3(x)\right)$$

$$e_w = -\frac{b}{4\lambda^2 EI}\int_0^x q(z)\left(\frac{H_1(x-z)}{\varphi_{31}} - \frac{H_3(x-z)}{\varphi_{32}}\right)\mathrm{d}z$$

$$e_\theta = \frac{b}{2EI}\int_0^x q(z)\frac{H_4(x-z)}{\varphi_{31}\varphi_{32}}\mathrm{d}z$$

$$e_M = -\frac{b}{2}\int_0^x q(z)\left(\frac{H_1(x-z)}{\varphi_{31}} + \frac{H_3(x-z)}{\varphi_{32}}\right)\mathrm{d}z$$

$$e_Q = -b\int_0^x q(z)\left[H_2(x-z) - \frac{\rho}{\sqrt{\rho^2-1}}H_4(x-z)\right]\mathrm{d}z$$