Appendix A. Solutions for different ground forms

The internal support pressure is denoted as P_{in}, with P_{rg} representing the pressure at the interface in natural and reinforced ground. The elastic modulus, Poisson ratio, the peak and residual cohesion, and the peak and residual friction angle in natural ground are designated as E_1, ν_1, c_1, $c_{1\mathrm{b}}$, φ_1 and $\varphi_{1\mathrm{b}}$, respectively, while those in the reinforced ground are defined as E_2, ν_2, c_2, $c_{2\mathrm{b}}$, φ_2, $\varphi_{2\mathrm{b}}$.

A.1 Form 4

The boundary conditions of Form 4 are:

$$\begin{cases} \sigma_r|_{r=r_0} = P_{\mathrm{in}};\ \sigma_r|_{r=R_1} = P_{R_1};\ \sigma_r|_{r=r_{\mathrm{g}}} = P_{r_{\mathrm{g}}} \\ \sigma_r|_{r=R_2} = P_{R_2};\ \sigma_r|_{r\to\infty} = P_0;\ u_r|_{r=r_{\mathrm{g}}+\mathrm{d}r} = u_r|_{r=r_{\mathrm{g}}-\mathrm{d}r} \\ \sigma_\theta - \sigma_r|_{r=R_1+\mathrm{d}r} = \sigma_\theta - \sigma_r|_{r=R_1-\mathrm{d}r} \\ \sigma_\theta - \sigma_r|_{r=R_2+\mathrm{d}r} = \sigma_\theta - \sigma_r|_{r=R_2-\mathrm{d}r} \end{cases} \tag{A-1}$$

where P_{R_1} and P_{R_2} are the radial stresses at $r=R_1$ and $r=R_2$, respectively.

According to Equations (2), (7) and (A-1), the stresses in different regions can be obtained as:

1. $r \geq R_2$:

$$\sigma_r = \frac{\left(P_{R_2} - P_0\right)R_2^2}{r^2} + P_0$$

$$\sigma_\theta = -\frac{\left(P_{R_2} - P_0\right)R_2^2}{r^2} + P_0$$

2. $r_{\mathrm{g}} \leq r \leq R_2$:

$$\sigma_r = \left(P_{\mathrm{rg}} + c_{1\mathrm{b}}\cot\varphi_{1\mathrm{b}}\right)\left(\frac{r}{r_{\mathrm{g}}}\right)^{K_{1\mathrm{b}}-1} - c_{1\mathrm{b}}\cot\varphi_{1\mathrm{b}}$$

$$\sigma_\theta = K_{1\mathrm{b}}\left(P_{\mathrm{rg}} + c_{1\mathrm{b}}\cot\varphi_{1\mathrm{b}}\right)\left(\frac{r}{r_{\mathrm{g}}}\right)^{K_{1\mathrm{b}}-1} - c_{1\mathrm{b}}\cot\varphi_{1\mathrm{b}}$$

3. $R_1 \leq r \leq r$:

$$\sigma_r = \frac{\left(P_{R_1} - P_{r_{\mathrm{g}}}\right)R_1^2 r_{\mathrm{g}}^2}{\left(r_{\mathrm{g}}^2 - R_1^2\right)r^2} + \frac{P_{r_{\mathrm{g}}} r_{\mathrm{g}}^2 - P_{R_1}R_1^2}{r_{\mathrm{g}}^2 - R_1^2}$$

$$\sigma_\theta = -\frac{\left(P_{R_1} - P_{\mathrm{rg}}\right)R_1^2 r_{\mathrm{g}}^2}{\left(r_{\mathrm{g}}^2 - R_1^2\right)r^2} + \frac{P_{\mathrm{rg}} r_{\mathrm{g}}^2 - P_{R_1}R_1^2}{r_{\mathrm{g}}^2 - R_1^2}$$

4. $r_0 \leq r \leq R_1$:

$$\sigma_r = \left(P_{\mathrm{in}} + c_{2\mathrm{b}}\cot\varphi_{2\mathrm{b}}\right)\left(\frac{r}{r_0}\right)^{K_{2\mathrm{b}}-1} - c_{2\mathrm{b}}\cot\varphi_{2\mathrm{b}}$$

$$\sigma_\theta = K_{2\mathrm{b}}\left(P_{\mathrm{in}} + c_{2\mathrm{b}}\cot\varphi_{2\mathrm{b}}\right)\left(\frac{r}{r_0}\right)^{K_{2\mathrm{b}}-1} - c_{2\mathrm{b}}\cot\varphi_{2\mathrm{b}} \tag{A-2}$$

Substituting $\sigma_{r|r=R_2}=P_{R_2}$ and $\sigma_{r|r=R_1}=P_{R_1}$ into Equation (A-2) yields

$$P_{R_1}=\left(P_{\text{in}}+c_{2\text{b}}\cot\varphi_{2\text{b}}\right)\left(\frac{R_1}{r_0}\right)^{K_{2\text{b}}-1}-c_{2\text{b}}\cot\varphi_{2\text{b}} \tag{A-3}$$

$$P_{R_2}=\left(P_{r_g}+c_{1\text{b}}\cot\varphi_{1\text{b}}\right)\left(\frac{R_2}{r_g}\right)^{K_{1\text{b}}-1}-c_{1\text{b}}\cot\varphi_{1\text{b}} \tag{A-4}$$

Equation (1) must be satisfied at $r=R_2$ and $r=R_1$. Therefore, the stress state $\sigma_\theta-\sigma_\text{r}|_{r=R_2+\text{d}r}=\sigma_\theta-\sigma_\text{r}|_{r=R_2-\text{d}r}$ and $\sigma_\theta-\sigma_r|_{r=R_1+\text{d}r}=\sigma_\theta-\sigma_r|_{r=R_1-\text{d}r}$ should be satisfied. Combining Equations (A-1) and (A-2) yields

$$P_{R_2}=\frac{2P_0-\sigma_{1\text{c}}}{\left(K_{1\text{p}}+1\right)}$$

$$P_{R_1}=\frac{\left(2P_{\text{rg}}-\sigma_{2\text{c}}\right)r_\text{g}^2+\sigma_{2\text{c}}R_1^2}{\left(K_{2\text{p}}+1\right)r_\text{g}^2-\left(K_{2\text{p}}-1\right)R_1^2} \tag{A-5}$$

By substituting Equation (A-1) into Equation (8), the displacement in elastic zones can be obtained as

1. $r\geq R_2$:

$$u=\frac{1+\nu_1}{E_1}\left[-\left[\left(P_{R_2}-P_0\right)R_2^2\right]/r+\left(1-2\nu_1\right)P_0 r\right]$$

2. $R_1\leq r\leq r_\text{g}$:

$$u=\frac{1+\nu_2}{E_2(r_\text{g}^2-R_1^2)r}\left[\left(1-2\nu_2\right)(P_{r_g}r_\text{g}^2-P_{R_1}R_1^2)r^2-\left(P_{R_1}-P_{r_g}\right)R_1^2r_\text{g}^2\right] \tag{A-6}$$

Combining Equations (4), (5), and (9) yields

$$\frac{\text{d}u_\text{r}}{\text{d}r}+\alpha\frac{u_\text{r}}{r}=\varepsilon_\text{r}^\text{e}+\alpha\varepsilon_\theta^\text{e} \tag{A-7}$$

Considering the boundary condition $u_{r|r=R_1}=u_{R_1}$ and $u_{r|r=R_2}=u_{R_2}$ based on Equation (A-6), and combining Equations (6), (A-2) and (A-7), the displacement in plastic zones can be obtained as

1. $r_\text{g}\leq r\leq R_2$:

$$u=u_{R_2}\frac{R_2^{\alpha_1}}{r^{\alpha_1}}+\frac{N_1}{K_{1\text{b}}+\alpha_1}\left(\frac{r^{K_{1\text{b}}}}{r_\text{g}^{K_{1\text{b}}-1}}-\frac{R_2^{K_{1\text{b}}+\alpha_1}}{r_\text{g}^{K_{1\text{b}}-1}r^{\alpha_1}}\right)\left(P_{\text{rg}}+c_{1\text{b}}\cot\varphi_{1\text{b}}\right)+\frac{M_1}{\alpha_1+1}\left(r-\frac{R_2^{\alpha_1+1}}{r^{\alpha_1}}\right)$$

2. $r_0\leq r\leq R_1$:

$$u=u_{R_1}\frac{R_1^{\alpha_2}}{r^{\alpha_2}}+\frac{N_2}{K_{2\text{b}}+\alpha_2}\left(\frac{r^{K_{2\text{b}}}}{r_0^{K_{2\text{b}}-1}}-\frac{R_1^{K_{2\text{b}}+\alpha_2}}{r_0^{K_{2\text{b}}-1}r^{\alpha_2}}\right)\left(P_{\text{in}}+c_{2\text{b}}\cot\varphi_{2\text{b}}\right)+\frac{M_2}{\alpha_2+1}\left(r-\frac{R_1^{\alpha_2+1}}{r^{\alpha_2}}\right) \tag{A-8}$$

where

$$\begin{cases} N_1 = \frac{1+\nu_1}{E_1}(1-\nu_1-\alpha_1\nu_1+\alpha_1 K_{1b}-\alpha_1\nu_1 K_{1b}-\nu_1 K_{1b}) \\ M_1 = \frac{1+\nu_1}{E_1}(-1+2\nu_1+2\alpha_1\nu_1-\alpha_1)c_{1b}\cot\varphi_{1b} \\ N_2 = \frac{1+\nu_2}{E_2}(1-\nu_2-\alpha_2\nu_2+\alpha_2 K_{2b}-\alpha_2\nu_2 K_{2b}-\nu_2 K_{2b}) \\ M_2 = \frac{1+\nu_2}{E_2}(-1+2\nu_2+2\alpha_2\nu_2-\alpha_2)c_{2b}\cot\varphi_{2b} \end{cases} \tag{A-9}$$

where $\alpha_1=(1+\sin\psi_1)/(1-\sin\psi_1)$, $\alpha_2=(1+\sin\psi_2)/(1-\sin\psi_2)$, $K_{1b}=(1+\sin\varphi_{1b})/(1-\sin\varphi_{1b})$, $K_{2b}=(1+\sin\varphi_{2b})/(1-\sin\varphi_{2b})$.

The continuous displacement $u_{r|r=r_g+dr}=u_{r|r=r_g-dr}$ can be given by

$$\frac{1+\nu_1}{E_1}\left[2(1-\nu_1)P_0-P_{R_2}\right]\frac{R_2^{\alpha_1+1}}{r_g^{\alpha_1}}+\frac{N_1}{K_{1b}+\alpha_1}\left(r_g-\frac{R_2^{K_{1b}+\alpha_1}}{r_g^{K_{1b}-1+\alpha_1}}\right)\left(P_{rg}+c_{1b}\cot\varphi_{1b}\right)+\frac{M_1}{\alpha_1+1}\left(r_g-\frac{R_2^{\alpha_1+1}}{r_g^{\alpha_1}}\right)$$

$$=\frac{(1+\nu_2)r_g}{E_2}\left[\frac{\left(P_{r_g}-P_{R_1}\right)R_1^2}{r_g^2-R_1^2}+(1-2\nu_2)\frac{P_{r_g}r_g^2-P_{R_1}R_1^2}{r_g^2-R_1^2}\right] \tag{A-10}$$

By combining Equations (A-4), (A-5), (A-8) and (A-10), the displacement u_t solution of the plastic zone in the reinforced ground can be obtained. By substituting $r=r_0$ into Equation (A-8), the total tunnel displacement u_t can be obtained. Note that the actual tunnel displacement u_{r0} is obtained by subtracting the initial radial displacement u_0 (caused by P_0) from the total displacement u_t, which can be given by

$$u_{r0}=u_{R_1}\frac{R_1^{\alpha_2}}{r_0^{\alpha_2}}+\frac{N_2}{K_{2b}+\alpha_2}\left(r_0-\frac{R_1^{K_{2b}+\alpha_2}}{r_0^{K_{2b}+\alpha_2-1}}\right)(P_{in}+c_{2b}\cot\varphi_{2b})+\frac{M_2}{\alpha_2+1}\left(r_0-\frac{R_1^{\alpha_2+1}}{r_0^{\alpha_2}}\right)-u_0 \tag{A-11}$$

The initial displacement u_0 can be obtained by substituting $P_{in}=P_0$ into the displacement solution of Form 1 in Appendix A.2.

A.2 Form 1

The displacement u_{r_0} at $r=r_0$ can be obtained as

$$u_{r_0}=\frac{1+\nu_2}{E_2}r_0\left[-\frac{\left(P_{in}-P_{r_g}\right)r_g^2}{r_g^2-r_0^2}+(1-2\nu_2)\frac{P_{r_g}r_g^2-P_{in}r_0^2}{r_g^2-r_0^2}\right]-u_0 \tag{A-12}$$

P_{r_g} is given by

$$P_{r_g}=\left\{2\left[E_1\left(1-\nu_2^2\right)r_0^2P_{in}+E_2\left(1-\nu_1^2\right)\left(r_g^2-r_0^2\right)P_0\right]\right\}/\left\{\left\{E_2\left(1+\nu_1\right)\left(r_g^2-r_0^2\right)+E_1\left(1+\nu_2\right)\left[r_0^2+\left(1-2\nu_2\right)r_g^2\right]\right\}\right\} \tag{A-13}$$

A.3 Form 2

The displacement u_{r0} can be obtained as

$$u_{r_0}=u_{R_1}\frac{R_1^{\alpha_2}}{r_0^{\alpha_2}}+\frac{N_2}{K_{2b}+\alpha_2}\left(r_0-\frac{R_1^{K_{2b}+\alpha_2}}{r_0^{K_{2b}+\alpha_2-1}}\right)\left(P_{in}+c_{2b}\cot\varphi_{2b}\right)+\frac{M_2}{\alpha_2+1}\left(r_0-\frac{R_1^{\alpha_2+1}}{r_0^{\alpha_2}}\right)-u_0 \tag{A-14}$$

where $u_{R_1}=\frac{1+\nu_2}{E_2}R_1\left[-\frac{\left(P_{R_1}-P_{r_g}\right)r_g^2}{r_g^2-R_1^2}+(1-2\nu_2)\frac{P_{r_g}r_g^2-P_{R_1}R_1^2}{r_g^2-R_1^2}\right]$.

R_1 and P_{r_g} can be obtained by Equations (A-3), (A-13) and (A-15).

$$\frac{2\left(P_{r_g}-P_{R_1}\right)r_g^2}{r_g^2-R_1^2}=\left(K_{2p}-1\right)P_{R_1}+\sigma_{2c} \tag{A-15}$$

where $\sigma_{2c}=2c_{2p}\cos\varphi_{2p}/(1-\sin\varphi_{2p})$.

A.4 Form 3

The displacement u_{r_0} at $r=r_0$ can be obtained by Equation (A-12). P_{r_g} can be obtained by Equations (A-4), (A-10) and (A-16).

$$2\left(P_0-P_{R_2}\right)=\left(K_{1p}-1\right)P_{R_2}+\sigma_{1c} \tag{A-16}$$

where $\sigma_{1c}=2c_{1p}\cos\varphi_{1p}/(1-\sin\varphi_{1p})$.

A.5 Form 5

The displacement u_{r0} can be obtained as

$$u_{r_0}=u_{r_g}\frac{r_g^{\alpha_2}}{r_0^{\alpha_2}}+\frac{N_2}{K_{2b}+\alpha_2}\left(r_0-\frac{r_g^{K_{2b}+\alpha_2}}{r_0^{K_{2b}+\alpha_2-1}}\right)\left(P_i+c_{2b}\cot\varphi_{2b}\right)+\frac{M_2}{\alpha_2+1}\left(r_0-\frac{r_g^{\alpha_2+1}}{r_0^{\alpha_2}}\right)-u_0 \tag{A-17}$$

where $u_{r_g}=\frac{1+\nu_1}{E_1}r_g\left[2\left(1-\nu_1\right)P_0-P_{r_g}\right]$.

P_{r_g} can be obtained by

$$P_{r_g}=\left(P_{in}+c_{2b}\cot\varphi_{2b}\right)\left(\frac{r_g}{r_0}\right)^{K_{2b}-1}-c_{2b}\cot\varphi_{2b} \tag{A-18}$$

A.6 Form 6

The displacement u_{r_0} can be obtained by Equation (A-17). Different from Form 5:

$$u_{r_g}=u_{R_2}\frac{R_2^{\alpha_1}}{r_g^{\alpha_1}}+\frac{N_1}{K_{1b}+\alpha_1}\left(r_g-\frac{R_2^{K_{1b}+\alpha_1}}{r_g^{K_{1b}+\alpha_1-1}}\right)\left(P_{r_g}+c_{1b}\cot\varphi_{1b}\right)+\frac{M_1}{\alpha_1+1}\left(r_g-\frac{R_2^{\alpha_1+1}}{r_g^{\alpha_1}}\right),\ u_{R_2}=\frac{1+\nu_1}{E_1}R_2\left[2\left(1-\nu_1\right)P_0-P_{R_2}\right].$$

Substituting $r=r_0$ into Equation (A-17) and subtracting u_0, the displacement u_{r_0} can be obtained. P_{r_g} can be obtained by Equations (A-4), (A-16) and (A-18).

Appendix B. Solutions for critical displacements

B.1 Critical displacement u_{12}

u_{12} is the transition condition between Forms 1 and 2, which can be obtained by substituting $R_1=r_0$, P_{r_g}, and $P_{in}=P_{12}$ into Equation (A-12) as:

$$u_{12}=\frac{1+\nu_2}{E_2}r_0\left[-\frac{\left(P_{12}-P_{r_g}\right)r_g^2}{r_g^2-r_0^2}+\left(1-2\nu_2\right)\frac{P_{r_g}r_g^2-P_{12}r_0^2}{r_g^2-r_0^2}\right]-u_0 \tag{B-1}$$

P_{12} and P_{rg} can be obtained by Equations (A-13), (A-15) and $P_{12}=P_{R1}$.

B.2 Critical displacement u_{13}

u_{13} is the transition condition between Forms 1 and 3, which can be obtained by substituting P_{r_g} and $P_{in}=P_{13}$ into Equation (A-12) as:

$$u_{13}=\frac{1+\nu_2}{E_2}r_0\left[-\frac{\left(P_{13}-P_{r_g}\right)r_g^2}{r_g^2-r_0^2}+\left(1-2\nu_2\right)\frac{P_{r_g}r_g^2-P_{13}r_0^2}{r_g^2-r_0^2}\right]-u_0 \tag{B-2}$$

P_{13}=P_{in} and P_{rg} can be obtained by Equations (A-16), (B-3) and $P_{r_g}=P_{R_2}$.

$$\frac{1+\nu_1}{E_1}\left[2\left(1-\nu_1\right)P_0-P_{R_2}\right]=\frac{1+\nu_2}{E_2}\left[\frac{\left(P_{r_g}-P_{13}\right)r_0^2}{r_g^2-r_0^2}+\left(1-2\nu_2\right)\frac{P_{r_g}r_g^2-P_{13}r_0^2}{r_g^2-r_0^2}\right] \tag{B-3}$$

B.3 Critical displacement u_{14}

u_{14} is the transition condition between Forms 1 and 4. In this case, the surrounding rock parameters of reinforced and natural ground need to satisfy a specific relationship, so that R_1=r_0 and R_2=r_g are satisfied at the same time. By substituting R_1=r_0, P_{r_g}, and P_{in}=P_{14} into Equation (A-12), u_{14} can be obtained as

$$u_{14}=\frac{1+\nu_2}{E_2}r_0\left[-\frac{\left(P_{R_1}-P_{r_g}\right)r_g^2}{r_g^2-r_0^2}+\left(1-2\nu_2\right)\frac{P_{r_g}r_g^2-P_{R_1}r_0^2}{r_g^2-r_0^2}\right]-u_0 \tag{B-4}$$

P_{14}=P_{in} and P_{r_g} can be obtained by Equations (A-15), (A-16), (B-3), $P_{r_g}=P_{R_2}$ and $P_{14}=P_{R_1}$.

B.4 Critical displacement u_{24}

u_{24} is the transition condition between Forms 2 and 4, which can be obtained by substituting R_2=r_g and P_{in}=P_{24} into Equation (A-11) as:

$$u_{24}=u_{R_1}\frac{R_1^{\alpha_2}}{r_0^{\alpha_2}}+\frac{N_2}{K_{2b}+\alpha_2}\left(r_0-\frac{R_1^{K_{2b}+\alpha_2}}{r_0^{K_{2b}+\alpha_2-1}}\right)\left(P_{24}+c_{2b}\cot\varphi_{2b}\right)+\frac{M_2}{\alpha_2+1}\left(r_0-\frac{R_1^{\alpha_2+1}}{r_0^{\alpha_2}}\right)-u_0 \tag{B-5}$$

P_{24}=P_{in}, P_{r_g}, P_{R_1}, and R_1 can be obtained by Equations (A-3), (A-15), (A-16), (B-3) and $P_{r_g}=P_{R_2}$. If the result shows R_1<0 or R_1>r_g, it indicates that u_{24} does not exist, and Form 2 will not transit to Form 4.

B.5 Critical displacement u_{25}

u_{25} is the transition condition between Forms 2 and 5. The interface of reinforced ground and natural ground is the elasto-plastic interface, so that $\varepsilon^{p}_{\theta|r=r_g}=0$ satisfies at r=r_g. Combining Equations (4), (6), (A-11) and (A-17) yields:

$$\varepsilon_\theta^{p}\Big|_{r=r_g}=\frac{1+\nu_1}{E_1}[2\left(1-\nu_1\right)P_0-P_{r_g}]-\frac{1+\nu_2}{E_2}\left[\left(K_{2b}-\nu_2K_{2b}-\nu_2\right)(P_{25}+c_{2b}\cot\varphi_{2b})\cdot\left(\frac{r_g}{r_0}\right)^{K_{2b}-1}-\left(1-2\nu_2\right)c_{2b}\cot\varphi_{2b}\right]=0 \tag{B-6}$$

Substituting P_{25}=P_{in} into Equations (A-18) and (B-7) yields:

$$P_{r_g}=\left(P_{25}+c_{2b}\cot\varphi_{2b}\right)\left(\frac{r_g}{r_0}\right)^{K_{2b}-1}-c_{2b}\cot\varphi_{2b} \tag{B-7}$$

By referring to Equation (A-11), u_{25} can be obtained as:

$$u_{25}=u_{r_g}\frac{r_g^{\alpha_2}}{r_0^{\alpha_2}}+\frac{N_2}{K_{2b}+\alpha_2}\left(r_0-\frac{r_g^{K_{2b}+\alpha_2}}{r_0^{K_{2b}+\alpha_2-1}}\right)\left(P_{25}+c_{2b}\cot\varphi_{2b}\right)+\frac{M_2}{\alpha_2+1}\left(r_0-\frac{r_g^{\alpha_2+1}}{r_0^{\alpha_2}}\right)-u_0 \tag{B-8}$$

B.6 Critical displacement u_{34}

u_{34} is the transition condition between Forms 3 and 4, which can be obtained by substituting R_1=r_0 and P_{in}=P_{34} into Equation (A-12) as:

$$u_{34}=\frac{1+\nu_2}{E_2}r_0\left[-\frac{\left(P_{34}-P_{r_g}\right)r_g^2}{r_g^2-r_0^2}+\left(1-2\nu_2\right)\frac{P_{r_g}r_g^2-P_{34}r_0^2}{r_g^2-r_0^2}\right]-u_0 \tag{B-9}$$

P_{34}=P_{in}, P_{r_g} , P_{R_1}, and R_1 can be obtained by Equations (A-4), (A-10), (A-15), (A-16) and P_{34}=P_{R_1} . If the result shows R_2<r_g, it indicates that u_{34} does not exist and Form 3 will not transit to Form 4.

B.7 Critical displacement u_{46}

u_{46} is the transition condition between Forms 4 and 6. In this case, plastic tangential strain at r=r_g is zero, i.e., $\varepsilon^{p}_{\theta|r=r_g}=0$ at r=r_g. Combining Equations (4), Eq. (6), and (A-8) yields:

$$\varepsilon_{\theta}^{p}\Big|_{r=r_g}=u_{R_2}\frac{R_2^{\alpha_1}}{r_g^{\alpha_1+1}}+\frac{N_1}{K_{1b}+\alpha_1}\left(1-\frac{R_2^{K_{1b}+\alpha_1}}{r_g^{K_{1b}+\alpha_1}}\right)\left(P_{r_g}+c_{1b}\cot\varphi_{1b}\right)+\frac{M_1}{\alpha_1+1}\left(1-\frac{R_2^{\alpha_1+1}}{r_g^{\alpha_1+1}}\right)-$$
$$\frac{1+\nu_2}{E_2}\left[\left(K_{2b}-\nu_2K_{2b}-\nu_2\right)\left(P_{in}+c_{2b}\cot\varphi_{2b}\right)\left(\frac{r}{r_0}\right)^{K_{2b}-1}-\left(1-2\nu_2\right)c_{2b}\cot\varphi_{2b}\right]=0 \tag{B-10}$$

P_{46}=P_{in}, R_1=r_g can be obtained by Equations (A-18), (A-16) and (A-4). By referring to Eq. (A-11), u_{46} can be obtained as:

$$u_{46}=u_{r_g}\frac{r_g^{\alpha_2}}{r_0^{\alpha_2}}+\frac{N_2}{K_{2b}+\alpha_2}\left(r_0-\frac{r_g^{K_{2b}+\alpha_2}}{r_0^{K_{2b}+\alpha_2-1}}\right)\left(P_{46}+c_{2b}\cot\varphi_{2b}\right)+\frac{M_2}{\alpha_2+1}\left(r_0-\frac{r_g^{\alpha_2+1}}{r_0^{\alpha_2}}\right)-u_0 \tag{B-11}$$

B.8 Critical displacement u_{56}

u_{56} is the transition condition between Forms 5 and 6, which can be obtained by substituting R_2=r_g into Equation (A-17) as

$$u_{56}=u_{r_g}\frac{r_g^{\alpha_2}}{r_0^{\alpha_2}}+\frac{N_2}{K_{2b}+\alpha_2}\left(r_0-\frac{r_g^{K_{2b}+\alpha_2}}{r_0^{K_{2b}+\alpha_2-1}}\right)\left(P_{56}+c_{2b}\cot\varphi_{2b}\right)+\frac{M_2}{\alpha_2+1}\left(r_0-\frac{r_g^{\alpha_2+1}}{r_0^{\alpha_2}}\right)-u_0 \tag{B-12}$$

By referring to Equation (A-11), the critical internal support pressure P_{in}=P_{56}, the stress P_{r_g} at r=r_g can be obtained by Equations (A-16), (A-3) and P_{R_2}=P_{r_g}.